The Oil Drum
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The Generalized Hubbert Curve

Posted by Gail the Actuary on October 26, 2010 - 10:20am
Topic: Supply/Production
Tags: geoffrey evans, steve mohr [list all tags]

This is a guest post by Steve Mohr and Geoffrey Evans. Steve is a recent Australian PhD. He and Geoffrey Evans also wrote Forecasting coal production until 2100. We also published a summary of Steve's thesis, Projection of World Fossil Fuel Production with Supply and Demand Interactions.

It is important to know the timing of when a resource will peak. It is ideal if a model of the likely production could be determined quickly, easily and accurately. The Hubbert Curve has been used frequently in the literature because it is quick, simple and, as shown with US oil production, accurate. But there is a problem, what to do with production statistics that do not conform to the Hubbert curve e.g. world oil production?

Solutions exist, but are not ideal, for instance, the disruption could be ignored and a Hubbert curve used anyway, but that tends to generate a poorly fitting model to the data. Alternatively a multiple Hubbert Curve technique could be used, but then the fitting is more complicated and the justification that production should follow the path becomes more difficult.

There is a solution: The Generalised Hubbert Curve[1], which is defined as:

$$\frac{dQ(t)}{dt} = rQ(t)\left(1-\frac{Q(t)}{Q_T}\right)x(t)$$

Where *Q(t)* is the cumulative production, Q_T is the URR and *r* is the rate constant, as per the Hubbert Curve, and *x(t)* is the intervention function. This can be solved to obtain:

$$\frac{dQ(t)}{dt} = \frac{rQ_T x(t)/2}{1+\cosh\left(-r\left(\int_0^t x(\tau)d\tau - t_p\right)\right)}$$

The intervention function is defined as:

$$x(t) = 1 + f_1(t) + \ldots + f_n(t)$$

where $f_i(t)$ represents a disruptions and is given mathematically as

$$f(t) = \begin{cases} 0 & t < t_{di} \\ -c_i\left(t - t_{di}\right)/t_{ri} & t_{di} \le t < t_{di} + t_{ri} \\ -c_i \exp\left(-b_i\left(t - t_{di} - t_{ri}\right)\right) & t_{di} + t_{ri} \le t \end{cases}$$

Trivially, with no disruptions we have $x(t) = 1$ and the Generalized Hubert Curve, becomes the usual Hubbert curve. An interesting aside, is to observe that the annual production of the Hubbert Curve and the annual production of the Generalised Bass Model are identical [1].

Perhaps an example will help. Let us use oil as the production data is well known and there were disruptions to production. Apply the following steps:

1) Fit a normal Hubbert Curve to the production statistics BEFORE the disruption occurred (<1974). Typically the URR Q_T is known from literature/your own judgment. For oil, let us assume it is 2234 Gb. r and t_p can be determined via Hubbert linearization or by eye, and we'll say they are 0.075 and 1998. From this we obtain Figure 1A:

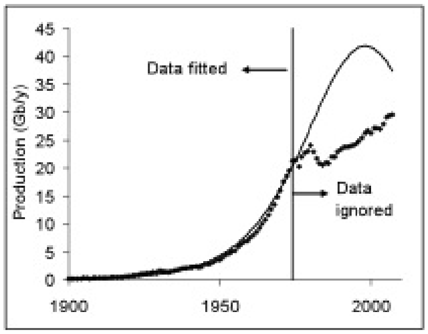

Figure 1A *Hubbert Curve*

2) Look at the first disruption fit t_{d1}, c_1, b_1 and tr1 to the production statistics during the disruption This is easier than it sounds: t_{d1} is known (~1974 first OPEC crisis), c_1 controls how far the production decreased, t_{r1} controls how long production was decreasing (1 year), and b_1 controls how quickly the production recovered (Figure 1B).

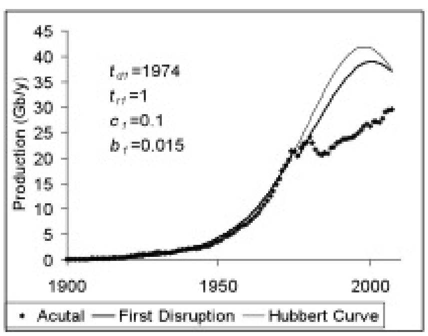

Figure 1B *First Disruption*

3) Repeat step 2 for the other disruptions.

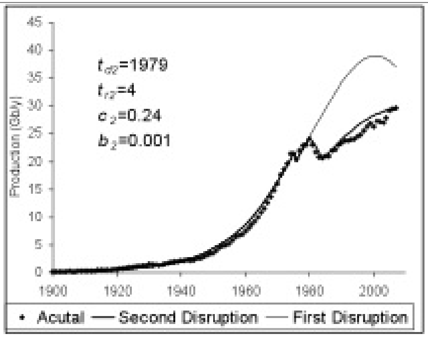

Figure 1C *Second Disruption*

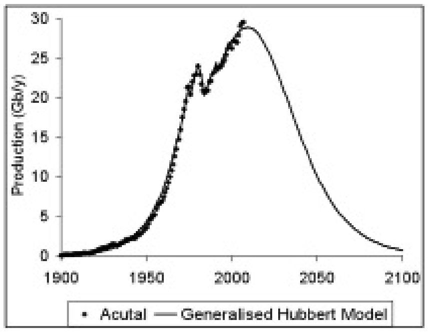

Figure 1D *The Generalised Hubbert Model*

If you're interested in trying out the model yourself, go to the attached Excel spreadsheet.

[1] Mohr, S. H. and Evans, G. M. "Combined Generalised Hubbert-Bass Model Approach to Include Disruptions When Predicting Future Oil Production" Natural Resources, 1(1) 28-33, 2010.