The Oil Drum
DISCUSSIONS ABOUT ENERGY AND OUR FUTURE

Energy Quality and Economic Value

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This is a guest post by Roger Brown, known as Roger K, whose graduate work was in physics. In reading about net energy and EROEI, he realized that energy balance alone is insufficient for characterizing the economics of energy production. In this post, he develops a multi-variable approach to account for the cost of other production resources. This post is the first publication of his innovative ideas. A summary is available at the end of the post.

Labor Cost of Energy

In order to produce an economic output, you have to invest production resources. At a minimum some amount of human labor must be invested. There is no such thing as a labor-free production process. Even if you lived in a sparsely inhabited tropical paradise filled with streams jumping with large tasty fish and heavily laden fruit trees growing profusely in the natural forests, you would still have to spend some amount of time gathering fruit and fish.

If you could gather all the food you needed for a single day in a half hour of work, then your food would be very cheap. If you lived in a less productive natural environment and had to spend eight hours a day gathering all of the food you needed, then your food would be very expensive.

Notice that a natural scale for labor costs exists due to the fact that, on average, the labor of one human being supports one human being. The smaller the percentage of total time that is spent gathering food, the lower the cost.

This natural scale may be inapplicable to a subset of individuals within a society if income inequality exists. To take an extreme case, suppose that an absolute monarch is served by an army of slaves. The slaves support themselves at a subsistence level by means of their work, and the king appropriates all of their excess productivity. The king's wealth depends upon the sum of all the excess output that he appropriates from his slaves.

If the army of slaves is large enough, the king could still be rich even if the excess wealth provided by each slave is small. That is the king could be wealthy even if the labor cost of production was high in the sense described above. However, for society as a whole the natural scale of labor cost still applies. Insofar as economic production depends primarily on labor, average income is high or low depending on the labor cost of producing economic output.

Of course other inputs besides labor may be important, but in many cases the cost of these inputs can be reduced to labor costs. Continuing the relatively simple example of a hunter-gatherer society, if wood is used for cooking, then the cost of that wood is the labor required to gather it. If wood is abundant in the locations where you are doing your hunting and gathering, then the labor cost will be low. If wood is relatively scarce then the labor costs will be high. If clay pots are used for cooking and eating, then the cost of production can be reduced to the labor cost of gathering clay, forming pots, and firing them. And so forth.

Special considerations arise when the input to the production process is the same as the output of the process. Consider gathering food. You have to burn calories in the process of obtaining calories. If the amount of calories burned exceeds some background rate of energy consumption, then the excess calories reduce the net benefit of the food gathering process.

To take a specific example suppose that, on average, one particular food gathering technique produces 1000 calories and burns 200 excess calories in an hours worth of effort. Then the net energy production per hour is 800 calories. Now suppose that a second food gathering process exists which burns only 100 calories per hour and produces 500 calories. In this case the net energy production per hour is only 400 calories. In spite of the fact that the second food gathering process has the same ratio of output energy to input energy as the first process it is more labor intensive, and therefore economically inferior to the first if labor is a limiting factor of production.

Of course energy balance plays a role in determining the labor efficiency of energy production. In order to quantitatively analyze this labor efficiency I introduce the following notation:

O = Output energy
P = Energy input to the production process
N = O - P = Net output energy
R = Non energy resource input to the energy production process (in this case labor hours)

The labor efficiency of energy production is the net energy produced divided by the labor hours expended:

Labor Efficiency = N/R.

This equation can be rewritten as:

[1] Labor Efficiency = $(N/O)/(R/O) = \mu/r$

Where $\mu = N/O$ and $r = R/O$. The variable μ is the fraction of the output energy which is left over after the input energy has been subtracted out. I call this number the energy utilization rate. It is the fraction of the output energy which is available to produce goods and services other than energy. The rest of the output energy must be diverted to producing a new batch of energy or the economy will come crashing down into ruins.

The variable r is the resource intensity (or resource cost) of gross energy production. It is the amount of resource (in the case given above, labor hours) that has to be expended to produce 1 unit of output energy. In the example given above both food gathering processes have $\mu=0.8$ but have different values of r (0.001 hours/calorie and 0.0005 hours/calorie respectively).

So far I have been talking about the case where labor is a limiting factor of production. If labor were not a limiting factor of production, might it not be true that energy balance is the only relevant factor in energy production? The answer to this question is no. Some non-energy resource is always relevant to the economics of energy production. Of course it is possible for other non-energy related factors of production such as supplies of fresh water or of arable land to be limiting factors of production, but the only case in which some non-energy related factor of production is not relevant to the economic quality of energy production is the case of energy production by magic.

For example, to return to the case of a sparsely populated tropical paradise, suppose than when you wake up in paradise each morning, manna from heaven sufficient for your day's nutritional needs has appeared on the ground beside you. This is truly free energy production and, as such, negates the need for economic analysis. If all that you need is provided without effort, then economics, defined as the practice of apportioning the use of scarce resources, does not exist. Even if you had to clap your hands once before the manna appeared, the difference between this process and absolutely free energy is too small of be of any practical importance.

If, on the other hand, every time you clap your hands a small amount of energy appears, and you need to clap your hands for several hours a day to produce a useful amount of energy, then economic analysis can be applied to this process. The net energy produced per hour of labor is an important parameter in analyzing the economics of this process. Yes, the energy balance matters. If clapping your hands burns up a significant amount of excess energy, then this energy must be accounted for in determining the productivity of this process, and it is accounted for by the factor μ in the above expression for the labor efficiency of energy production.

Energy and Labor Optimization of Economic Output

Now I wish to discuss the optimization of the apportionment of a scarce resource in economic production, and in particular, the apportionment of labor. Therefore I will consider an economic system in which the only limiting factor of production apart from energy is labor. One might object that this is not a realistic case since raw materials are always required in addition to energy and labor. However, labor is always limited since, on average, the labor of one human being supports one human being, whereas the supply of raw materials could be effectively infinite.

Even if you had an endless copper mine, the amount of time it takes to extract and smelt a given amount of ore limits how much copper you can produce in a given period of time. So my assumption is that raw materials are available in large abundance relative to any short to intermediate term use we might have for them and the amount we produce is limited by how much labor we choose to expend in extracting them.

I will also assume that energy production is only limited by the amount of labor dedicated to the energy extraction process. If we dedicate a larger fraction of our labor hours to extracting energy our net energy increases, but the number of labor hours available for utilizing this energy in the manufacture of useful products and services decreases. In some sense labor is the only real cost in this system. We do not give economic output to coal deposits or to deposits of metal ore. We give economic output to people who extract useful outputs from these deposits.

Of course in the real world we do give economic output to the 'owners' of natural resources in the form of economic rent, and we give money to the providers of capital in the form of interest. Such costs are not labor costs. However, I would argue that rent and interest are political costs and not real physical costs. That is to say that no physical reason exists why rent and interest have to be paid in order produce economic output. Of course, I have been assured over and over again by a variety of people that any attempt to eliminate rent and interest will inevitably result in the creation of a monstrous, inefficient, socialist bureaucracy. This claim may or may not be correct, but at present I am inquiring into the physical basis of wealth so that I will ignore these non-physical costs of producing wealth.

To be concrete suppose that you are alone on an island and are therefore producing all of your own economic output. In this case no one exists to receive payments of rent and interest. A fuel source is available that provides μ/r units of energy per hour of labor with no limit on how much energy can be obtained other than the number of hours worked. The question which I wish to enquire into is what fraction of total labor time should be dedicated to producing energy and what fraction should be dedicated to using that energy to produce useful goods and services.

In order to answer this question we must consider the use of technology in the process of economic production. The primary function of technology is to improve the productivity of labor. A hunter armed with bows and arrow will gather more meat per hour of hunting than one armed with a spear. A root gatherer armed with a digging stick will collect more kilograms of roots per hour than one using only his or her hands. Wheeled carts will allow a human being to transport a larger amount of mass per hour than ordinary human locomotion. And so forth.

Adding external energy sources to the mix further improves labor productivity. If a horse is used to transport the hunter, he can cover more ground in a given period of time and take more game. If an ox is attached to a wheeled cart then the amount of human effort required to transport a given mass of material drops dramatically. The use of fossil fuels did not qualitatively change the direction technological improvement to economic production. Their use merely accelerated the rate of improvement in productivity.

In my view it is impossible to separate productivity into separate contributions made by labor, tools, and energy. There is no such thing as a labor free process. No matter how automated the machinery used, someone has to build and maintain the machines, extract and deliver the energy which runs the machines, and so forth. Energy does not 'replace' labor; It creates a more productive synergy of labor and tools. Properly speaking one can only speak of the productivity of a given economic synergy and not the productivity of the individual components of that synergy. Labor productivity can be used as a metric, but labor productivity cannot really be separated from the overall productivity of all inputs to the production process.

In order to discuss this productive synergy I will introduce a concept which I call the average productivity function P. I define P as the average productivity per hour of labor in the non-energy producing sector of the economy. I will assume that $P=P[E]$ where E is the energy per worker hour available in the non-energy producing sector of the economy. What form might the function $P[E]$ take? One possibility is that it is a linear function of E. In this case, if I double the energy per worker hour I double my productivity. If I triple the energy per worker hour I triple my productivity, and so forth. It is easy to show, however, that if productivity increases linearly with E forever, then an absurd conclusion will be reached.

Suppose that I work a total of H hours and I spend a fraction f of those hours obtaining energy and fraction 1-f using that energy to provide useful goods and services. The net energy available for the production of useful goods and services is:

Net Energy = $H \times f \times \mu / r$

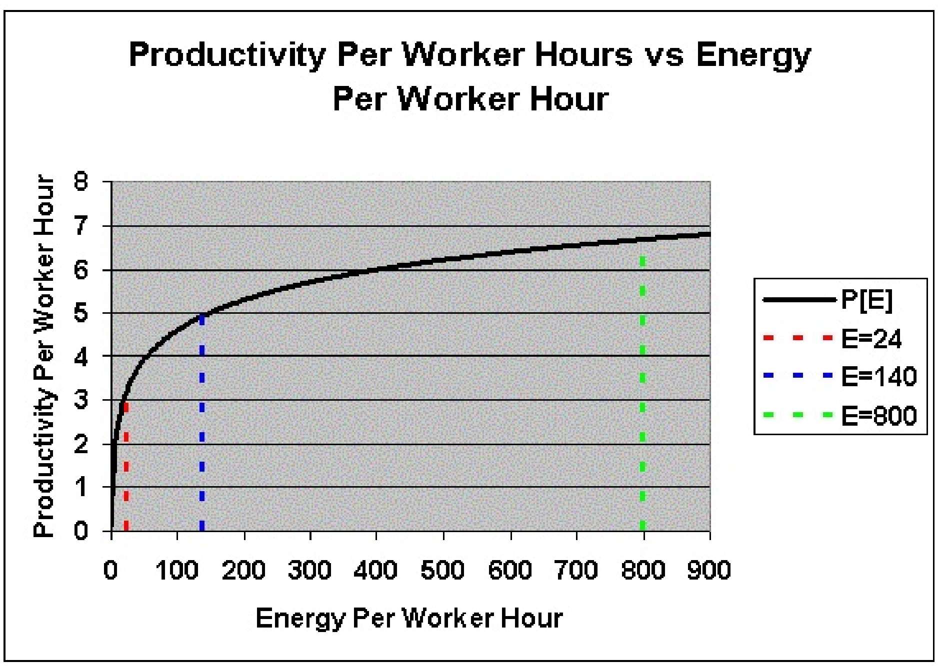
Since the hours worked in the non-energy producing portion of the economy are $H \times (1-f)$ the energy per worker hour is given by:

[2] Energy/Hour = $E = (\mu/r) \times f/(1-f)$

If the average productivity is linearly proportional to $C \times E$ (where C is a constant), then the total production will be proportional to $H \times (1-f)$ times this number:

Total Production = $C \times H \times (\mu/r) \times f$

From this equation we see that total production increases with f without having a maximum. That is, if I worked at harvesting fuel all year along until one hour before I knocked off for my New Year's Eve celebration, I would be more productive in the remaining hour than if I had stopped gathering fuel at any earlier time. This conclusion is obviously absurd. One can reach the same conclusion by considering a factory that employs 1000 people. If 999 people are fired the remaining employee has 1000 times as much energy available per hour as he or she did before the layoff, so that under the assumption of a linear relation between energy per hour and productivity one employee could match the output of 1000 employees. Therefore a strictly linear relation between energy/hour and productivity is not possible. As energy use per hour increases, the marginal return on additional energy use per worker hour must decrease. The graph below depicts qualitatively the general form that P[E] must take:



I have depicted P[E] going to zero as E goes to zero. One could also have P intersect the E=0 axis at a positive value corresponding to some background level of productivity obtainable with human muscle power only.

The E axis has the same units as μ/r, the net energy produced per hour of labor in the fuel producing process. The units of this axis are not important as one can define a new energy/hour scale by setting E'=k×E where k is a constant. Any point on the P[E] curve can be reached for any energy source with a positive value of μ/r by simply making the hours spent in the non-energy producing part of the economy arbitrarily small. However, at each point on the P[E] curve, only one value of μ/r makes the total productivity a maximum at that point. That is to say that for a given function P[E] of the form shown, there is an optimum operating point for an energy source of a given quality as measured by μ/r.

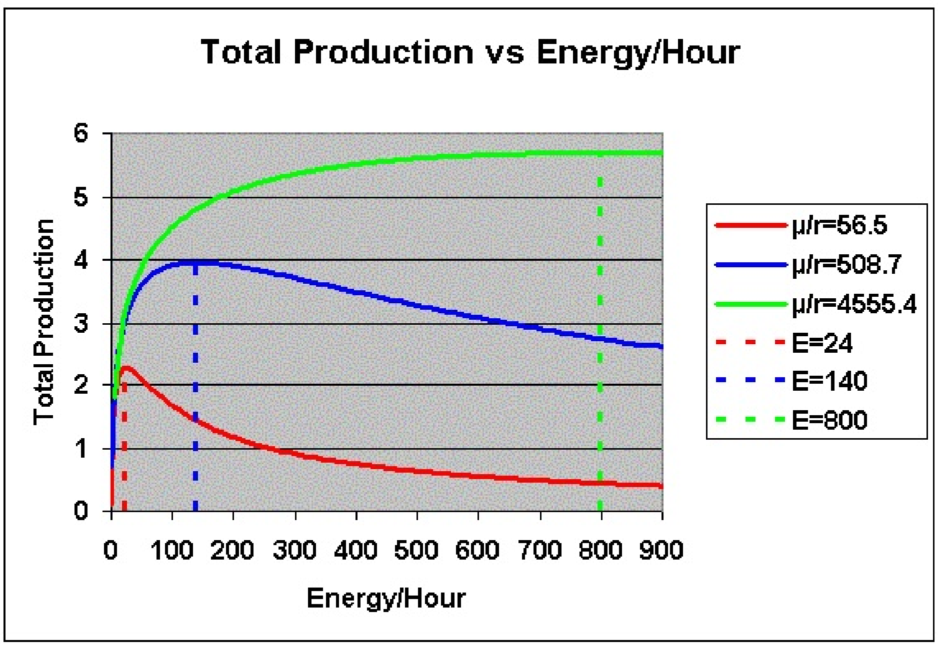
The total production will equal P times the hours spent in the non-energy producing portion of the economy. That is the total production is given by:

Production =H×(1-f)×P

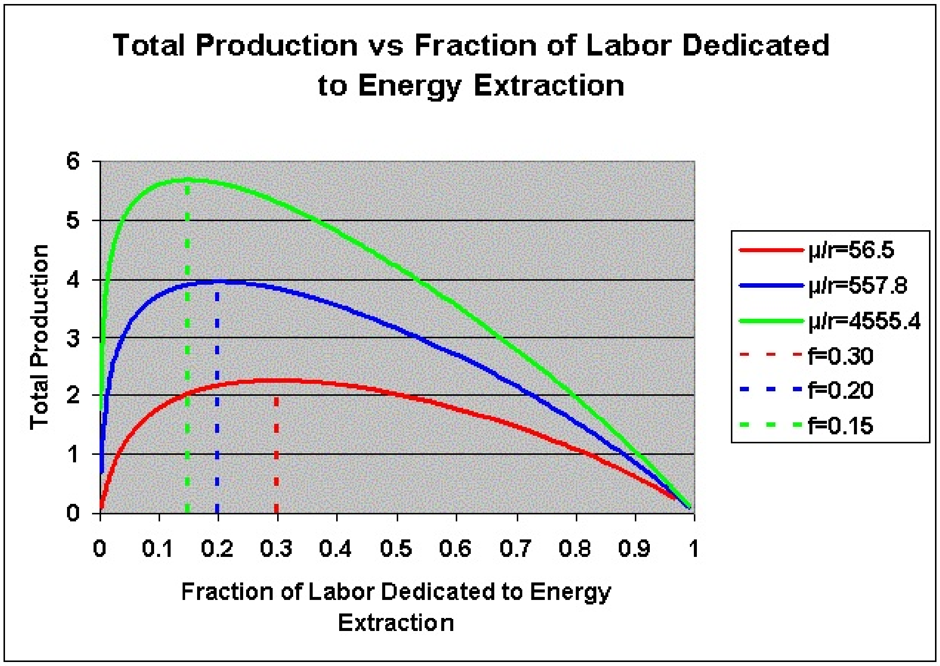
From the expression for E given above (equation [2]) we can solve for f in terms of μ/r:

f = E/(E+μ/r)

Therefore P[E]×(1-f) can be expressed as a function of E. The particular function given is P[E]=ln(E+1). On the following graph I show the P[E]×(1-f) vs. E for three different values of μ/r. I have chosen the values of μ/r so that the maxima correspond the E=24, 130, and 800 (The same values marked in the previous figure).

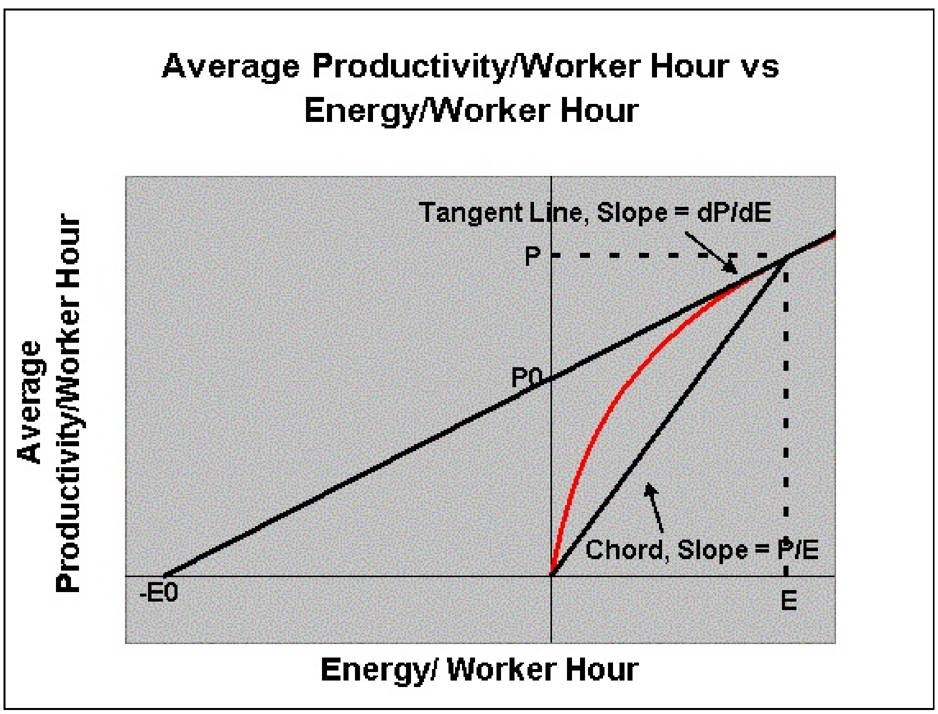


Note how slowly the maximum total production drops off compared to the energy per worker hour E. This slow drop-off is a consequence of the shape of the productivity curve P[E]. Of course the function P[E]=ln(E+1) is just an arbitrary function that has the right qualitative behavior, so that no quantitative conclusion can be reached from this particular example. However, a more general result can be derived from the differential geometry of the P[E] curve without knowledge of the complete function P[E]. In order to derive this result it is convenient to regard P[E] as a function of f via the equation E = (μ/r)×f/(1-f). In figure 3 I show P×(1-f) as a function of f for the same three values of μ/r.



It can be seen that the three values of E chosen correspond to f=0.15, 0.20, and 0.30 respectively. That is as the quality of the energy source (as measured by μ/r) decreases then the fraction of labor dedicated to extracting fuel increases.

Again this example depends on the specific properties of the function ln(E+1). A general result for the values of μ/r and f which maximize the total productivity = P×H×(1-f) can be derived by taking the derivative with respect to f and setting it equal to zero. The total hours worked is constant, so that maximizing P×(1-f) is sufficient to solve the problem. I give the proof in an appendix and here I merely state the results making reference to the following figure:



At a given point on the P[E] curve the value of μ/r which maximizes the total production is given by:

$\mu/r = [P/(dP/dE)]-E$

This equation can be rewritten as:

$(dP/dE)\times[(\mu/r)+E] = P$

Inspection of the figure shows that μ/r is equal to minus the intersection of the tangent line with the E axis:

$\mu/r = E0$

The fraction of labor dedicated to fuel extraction which, combined with the value of μ/r given above, maximizes production at the given point on the P[E] curve is given by:

$f = (dP/dE)/(P/E)$

This number is the ratio of the slope of the tangent line to the slope of the chord from the origin. The chord from the origin corresponds to a linear increase in productivity/hour with energy/hour. As I showed previously, with such a linear dependence labor productivity can be increased without bound so that f would have no maximum. The above result shows that the maximum occurs when f is equal to the ratio of the marginal increase in productivity with increasing E to the average increase in productivity with respect to E.

The equation for f can be rewritten as follows:

$f = [E\times(dP/dE)]/P$

Inspection of the figure shows that $E\times(dP/dE)$ is equal to P-P0 where P0 is the intersection of the tangent line with the P axis. Thus f is given by:

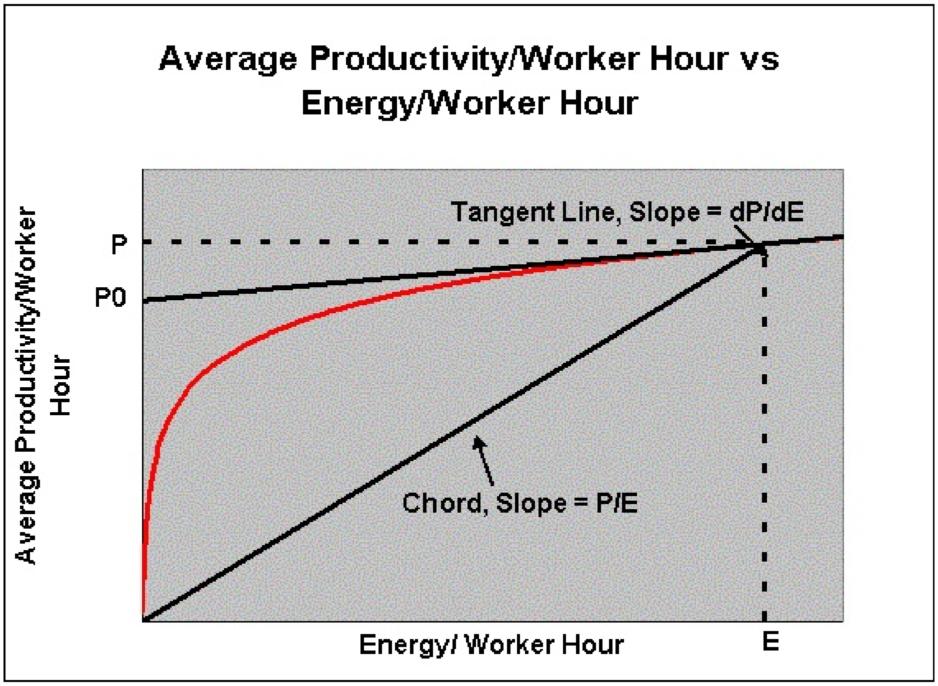
$f = (P-P0)/P = 1-(P0/P)$

The total production achieved at this operating point is:

$\text{Production} = H\times P\times(1-f) = H\times P0$

$H\times P$ is the total production that would be achieved at the given value of E if free energy were available so that all labor effort could be dedicated to producing useful goods and services. Therefore P0 is the effective productivity per hour achieved for all labor hours including those dedicated to producing fuel.

The following figure shows the tangent line and chord to the P[E] curve for the case that f=0.15.



The above results make it clear that within the parameters of this simplified economic model, the characteristic of a high quality energy source is that a small amount of effort spent gathering fuel drives the economy to a point where the marginal return on further energy use is small. Without such a small marginal return on additional use of energy, total production could be further increased by dedicating more resources to further energy extraction. This conclusion is independent of knowing the explicit form of the P[E] function. Thus the idea that that we could substantially back off our energy use while maintaining reasonable levels of productivity is supported by this model.

Up to now I have been talking as if the labor efficiency of energy production were an unchanging constant, and as if the productivity function P[E] was a fixed and eternal mathematical form. Neither of these statements is correct. In the early days of fossil fuel use the labor efficiency of fuel production was being increased steadily both through the discovery of newer higher quality reservoirs of fuel and through advancing extraction technology. The productivity function P[E] also depends on the quality of non-energy resources available (ore grades, etc.) as well as on the general advancement of technological knowledge, on the quantity and variety of the existing set of physical tools, and on the built up infrastructure of society generally.

The economic problem that we are faced with now is a potentially large degradation in the quality (e.g. higher labor and other resource intensities and lower energy balance) of the energy sources available to us and potentially a degradation in the quality of other resources as well (e.g. the necessity of using lower grade metallic ores). These changes will put a large burden on our technological prowess to maintain increasing levels of productivity. Some people claim that in the era of cheap energy, we were careless about energy use, so that a lot of low hanging fruit exists with respect to improvements in energy efficiency. If our economic goal is to produce physically and psychologically healthy human beings with a minimum consumption of resources, this claim may well be true. But if our goal is to produce growth in the traditional economic sense, this conclusion is questionable.

The above analysis suggests that even in the days of cheap energy, energy use was limited by the marginal utility of energy, so that a high motivation for increasing the efficiency with which energy is converted into economic value has always existed. If oil is such a unique, valuable, possibly irreplaceable resource, then why have we been using it to tool down the freeway at high speed encased in tons of metal? If that same energy could have produced a lot more value in some other part of the economy, why didn't it naturally flow there? The answer to this question may well be that in a culture in which the lawn, jet airplane tourism, home theater systems, etc. have become part of normal life, SUVs and luxury cars are the highest value marginal use of that form of energy that we could come up with. Yes, as energy prices rise, the most marginal uses will disappear first. It is far from clear, however, that we will necessarily be able to produce equal or larger value elsewhere in the economy.

In any event, it is clear that in a growth oriented economy we will always push out along the curve of marginal utility as swiftly as we can. No matter how clever we are at increasing our manufacturing efficiency, the tendency of the economy will be to push up against the limits of available energy resources as fast as possible.

If on the other hand, if we were to decommit from the goal of continuous growth and instead try to maintain a given quality of life with a minimum consumption of resources, then the combination of backing off on total per capita energy use along with efficiency improvements might allow us to maintain a decent quality of life even in the face of decreasing energy quality.

Multiple Resource Costs of Energy Production

I will now consider the role of other resources than labor used in the production of energy. I now suppose that multiple non-energy resources R_0, R_1,R_2 ..., R_n are input to an energy production process that outputs a total amount of energy O. As before I will assume that the energy input is P so that the net energy output is N=O-P. I will assume that R_0 is labor since no such thing as a labor free production process exists. If the other non-energy inputs have labor and energy embedded in their production, I assume that these inputs are included in P and in R_0.

You might ask if the embedded labor and energy have been subtracted from the input of a given resource is there any cost left over? In some cases the answer is no, while in other cases opportunity costs exist that are not captured in the embedded energy and labor. For example we cannot create high quality soil and arable land merely by the application of energy and labor, and since the supply of land and soil are finite there are opportunity costs associated with using these resources to produce energy rather than food.

As another example, consider the water required to process oil shale in the relatively arid American west. There are opportunity costs associated with using the available supply of water to produce fuel that cannot be captured by the labor and energy costs of delivering the water to the shale processing site.

I mentioned embedded energy above, and before proceeding with an analysis of the benefit to cost ratio of energy production in the case of multiple input resources, I want to introduce a concept which I have found helpful in clarifying the role of embedded energy. I call this concept the *working reserve of energy*. In order to run our economy effectively we need piles of coal, pipelines full of natural gas, tanks of refined petroleum fuel, stockpiles of uranium fuel rods, etc sitting around available for use by various economic producers. Energy producers pull fuel out of these stockpiles, but unlike other producers they also put fuel back into these same stockpiles, which I call the working reserves of energy.

In order for the economy to keep running on an ongoing basis the energy producers must keep the reserve full. That is they must not only replace the fuel that they themselves remove but also the fuel removed by all other economic producers. Withdrawals of fuel by energy producers can be direct or indirect. That is they can directly purchase fuel to run their machinery, or other producers can draw out fuel and create output which they sell to the energy producers for use in the fuel extraction process. This indirect withdrawal of fuel from the working reserves is embedded energy.

So now let us consider the benefit to cost ratio of producing energy under the simple assumption of the economic equivalence of all forms of energy which is generally (though incorrectly) used for energy balance calculations. Since multiple input resources are involved we need some method of converting resource quantities into units of a universal scale of value. Whether it is truly possible to objectively define such a scale in the real world may be doubted, but we cannot make progress in economic analysis without assuming that it exists. Therefore I will assume that the value of a unit of energy is Ve and the opportunity cost of dedicating one unit of a non-production resource Ri to the energy production process is Ci.

The benefit of energy production in our universal scale is then given by:

$$\text{Benefit} = Ve \times (O-P) = Ve \times N$$

Where O is the gross output energy, P is the energy consumed in the production process (both directly and indirectly) and N=O-P is the net energy produced. You might try to argue that Ve×O is the benefit of the energy production process, but this claim is false. The very last time you run an energy production process you can claim the whole output as a benefit, since you do not have to reinvest any of the energy to produce more energy. But the extra energy P that you get from this last batch is balanced by the energy P that you had to beg, borrow, or steal to process your first batch. In between, the production energy used in processing any particular batch of energy is not available to the rest of the economy.

The cost of producing the net energy N is given by:

$$\text{Cost} = C_0 \times R_0 + C_1 \times R_1 + \ldots + C_n \times R_n$$

The opportunity costs Ci are not the same as market prices. For one thing embedded labor and energy cost have been subtracted out (except for C_0 which is the opportunity cost of dedicating a unit of labor to the energy production process). As I pointed out previously, a finite resource such as land or fresh water may have opportunity costs associated with their use which are not captured in the embedded labor and energy.

Note that input energy is not included as part of the cost. It subtracts from the benefit but does not add to the cost. This claim may require some further explanation. As I mentioned previously, when the first batch of energy is produced from a new energy production process the input energy P must obtained from somewhere. Either some energy producer must process some extra energy, or the use of energy elsewhere in the economy must be curtailed. This initial input of energy is a cost. However, after the first batch of energy is processed and the output is placed in the working reserves, the energy producer has effectively provided the input for his or her next batch of energy. (I am assuming a positive energy balance.) For succeeding batches of energy no other energy producer has to provide extra energy. No other economic process has to forego the use of energy. Finally, when the last batch of energy is processed the initial input energy is recovered. Therefore society has incurred no net energy cost from the existence of this process.

The benefit to cost ratio is given by:

$$\text{Benefit/Cost} = (Ve \times N)/(C_0 \times R_0 + C_1 \times R_1 + \ldots + C_n \times R_n)$$

Dividing the numerator and denominator of this expression by the gross output energy O gives an interesting expression for the benefit to cost ratio:

$$\text{Benefit/Cost} = (Ve \times N/O)/(C_0 \times R_0/O + C_1 \times R_1/O + \ldots + C_n \times R_n/O)$$

I rewrite this equation as:

$$\text{Benefit/Cost} = (Ve \times \mu)/(C_0 \times r_0 + C_1 \times r_1 + \ldots + C_n \times r_n)$$

Where $\mu = N/O$ and $r_i = R_i/O$. Clearly μ is the energy utilization rate discussed earlier. The values $r_i = R_i/O$ I call the resource intensities or resource costs of gross energy production. The value r_i represents the amount of resource required to produce 1 output unit of energy. The values μ and $r_0, r_1, \ldots, r_n$ are physical parameters that depend on the nature of the energy producing process. In particular r_0 is the labor intensity of energy production which I discussed previously. The parameters Ve, C_0, C_1, ...,C_n are not physical parameters and are not constants. They are complex functions of the operation of the entire economy, among other things depending on the total scale of the economy and on the quality of natural resources available. Nevertheless if one takes a snapshot of the economy at a particular instant of time Ve, C_0, C_1, ...C_n may be regarded as approximately constant with respect to the extraction of a marginal unit of net energy and may therefore be used (assuming one really knew how calculate them) to estimate the benefit to cost ratio.

Note that with respect the physical parameters of energy production, two different (though often related) effects can lower the benefit/cost ratio. The energy balance as represented by the energy utilization rate μ rate can go down, or the resource intensities of energy production can go up. Often times both effects occur at the same time. Oil produced from oil shale has a lower energy balance than conventional oil and the labor and fresh water intensity of producing a barrel of oil from this source is much higher than for conventional oil.

In order for an energy production process to provide any economic benefit the benefit/cost ratio must be greater than 1, which implies that:

$$(C_0 \times r_0 + C_1 \times r_1 + \ldots + C_n \times r_n)/Ve < \mu$$

The left side of this equation is the ratio of the non-energy cost of producing a unit of energy to the value of the unit of energy so produced. Since not all of the energy produced is available for the production of useful goods and services the upper limit on this cost ratio is μ rather than 1. If this ratio is much smaller than 1.0, then very small values of μ can be economically justified. I call this limit the *magic wand limit* of energy production. If large quantities of energy could be harvested with trivial expenditures of labor and other non-energy related resources, then net economic value could be produced even with very small energy balances.

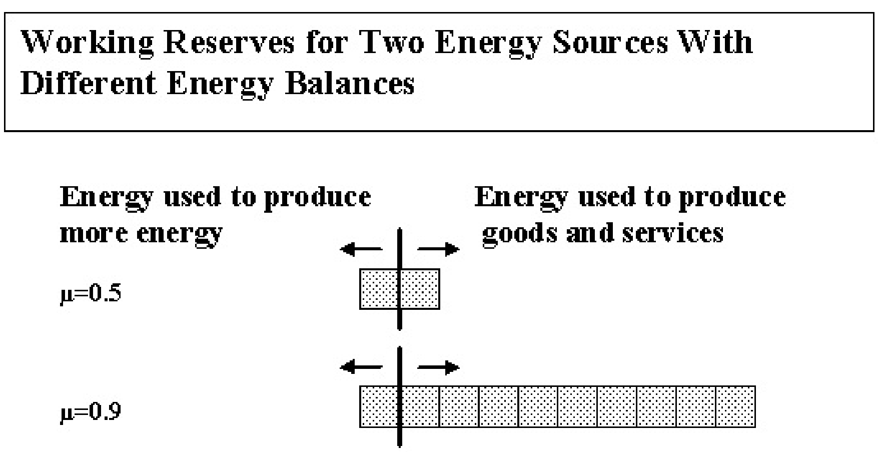
On the other hand, if this ratio is >=1 then the energy production process in question is economically useless no matter how good the energy balance may be, since the value of the non-energy resources consumed is equal to or greater than the value of the energy produced. For example, suppose you found a magical energy crop which grew and harvested itself without human intervention. The energy rich sap of some species of tree flowed out of the trees through underground channels and filled a natural reservoir. You go to the reservoir once a year and find that it is full of fuel. If dedicating that land to energy production meant that you would starve to death, then you would not give a damn about the perfect energy balance (or the infinite EROEI if you prefer).

Remember that the opportunity costs Ci are not constants. For example, if you use currently fallow land to produce fuel the opportunity cost of dedicating that land to energy production could be close to zero. On the other hand, if biofuel production is ramped up to the level where a large fraction of farm land is being dedicated to the production of fuel, then the opportunity cost of dedicating additional land to producing fuel could become quite high, so that even if the physical aspects of energy production as measured by the energy utilization rate μ and the land intensity of energy production remain constant, the benefit to cost ratio may drop significantly.

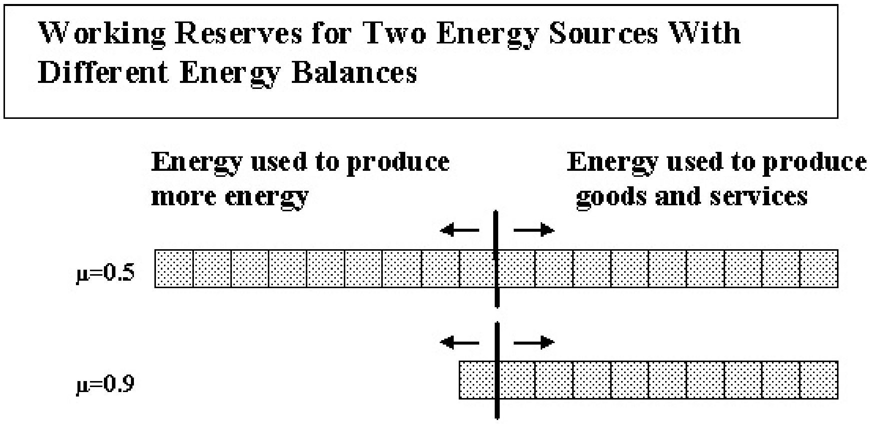
As another example, consider the labor cost of producing oil from tar sands. I have read that in Canada general labor costs are rising as tar sands production rises, so that people trying to hire retail clerks or food service personal are having to pay higher wages because of the competition from energy production. On average the labor of one human being supports one human being so, that if the fraction of total labor dedicated to energy extraction becomes significant, it can affect overall economic production.

Resource Costs of Energy Production and the Overall Scale of the Economy

Finally I want to discuss in more detail the relation of energy balance to the overall scale of the economy mentioned at the end of the last section. In this discussion I will make use of the concept of the working reserve of energy which I introduced earlier. The figure below shows hypothetical working reserves of energy for two economies run off of two different fuel sources with different energy balances (μ=0.5 and μ=0.9 respectively). I have represented the size of the reserves by the total area of a series of squares. The energy to the left of the vertical bar is used in the energy extraction process (directly or indirectly). The energy to the right of the vertical bar is used to produce useful goods and services. Energy producers naturally put energy into the reserves as well as taking it out. At the end of the effective lifetime of the reserves they are still full because the energy producers have put back all of the energy that was taken about by all varieties of producers.



If we assume the that working reserves of both fuels are intended to represent energy available for the same time period, then the second energy source provides nine times the net energy of the first energy source in the same time period. Can we therefore conclude that the relative economic value of these energy sources is in a ratio of nine to one? The answer to this question is not straightforward. The next figure shows the working reserves for the same two energy sources for the case where they are both providing the same amount of net energy. In order to get from case 1 to case 2 the size of the working reserves and the total rate of fuel production for the energy source with μ=0.5 has been expanded by a factor of 9.



Naturally this large expansion in energy production could not take place over night. It would take a substantial period of time to grow energy production by this amount. In order to achieve such growth one would have to dedicate to energy production a fraction of the total energy larger than the energy utilization rate μ (=0.5). So if growth is referred to a yearly time period, then dedicating 54% of the total available energy to the production of more energy would result in an energy production growth rate of 4% per year. As energy production grew the total non-energy resource cost of energy production would also grow. That is, if labor, land, and water are being used to produce energy, we would have to continuously expand the amount of these resources being dedicated to this process.

If there are negative externalities associated with energy production and consumption these are also growing as energy production grows. These resource costs and the negative externalities determine the economics of a given fuel source and not the disappearance of energy during the production process per se. Of course, the energy balance does affect these resource costs since, all other things being equal, a smaller energy balance implies a larger total energy extraction and a larger resource cost to produce the same amount of net energy.

The resource cost multiplication factor can be derived as follows. Recall that the cost of producing energy is given by:

$$Cost = C_0 \times R_0 + C_1 \times R_1 + \ldots + C_n \times R_n$$

Where R_0 is the labor input and R_i are other resources whose costs cannot be reduced to the embedded labor and energy. We are interested in the cost per unit of net energy produced so we divide by N and do a bit of algebra:

$$Cost/N = (O/N) \times \{C_0 \times R_0/O + C_1 \times R_1/O + \ldots + C_n \times R_n/O\}$$

Recalling that $R_i/O=r_i$ (the resource intensity of energy production) we find:

$$Cost/N = (O/N) \times \{C_0 \times r_0 + C_1 \times r_1 + \ldots + C_n \times r_n\}$$

The quantity in curly brackets is the cost of production of 1 unit of gross output energy, so that the required cost multiplication factor due to the energy balance is (O/N) = 1/μ. For the two energy sources depicted in the above figure the values of (O/N) are 2.0 and 1.11 respectively.

The ratio of these two numbers is the same as the ratio of the size of the required working reserves.

In general, a fuel source with a low energy balance will be more likely to run into problems of scale than one with a high energy balance as energy production is ramped up to produce a given amount of net energy. The real problem with biofuels is not low energy balance per se, but the large requirements of land, water, and soil as production is ramped up to high levels. Of course, the real cost of the scaling up a given energy source cannot be determined in any simple way from the energy balance. This cost depends on many complex interactions in the total economic system. Economics cannot be reduced to calculating energy ratios.

Appendix: Derivation of the values of labor efficiency of energy production (=μ/r) and of the fraction of labor dedicated to energy production (=f) which maximize the total productivity

We wish to maximize the following function:

$$P \times (1-f)$$

Taking the derivative with respect to f and setting it equal to zero we find:

$$\frac{dP}{dE} \times \frac{dE}{df} \times (1-f) - P = 0$$

We rewrite this equation as:

[1] $$P = \frac{dP}{dE} \times \frac{dE}{df} \times (1-f)$$

I have shown previously that the energy per worker hour E is given by:

[2] $$E = \frac{\mu}{r} \times \frac{f}{1-f}$$

Taking the derivative with respect to f we find:

[3] $$\frac{dE}{df} = \frac{\mu}{r} \times \frac{1}{(1-f)^2}$$

From equation [2] we see that:

[4] $$\frac{\mu}{r} = E \times \frac{1-f}{f}$$

Substituting this expression for μ/r into equation [3] we find:

[5] $$\frac{dE}{df} = E \times \frac{1}{f \times (1-f)}$$

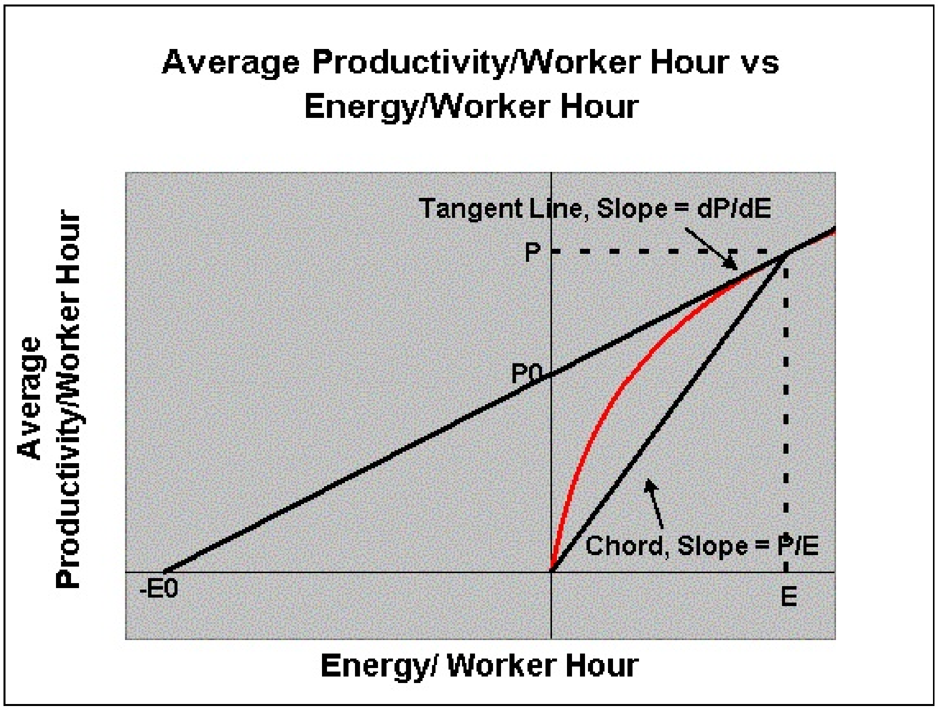
If we substitute equation [5] into equation [1] and multiply by f we find:

$$P \times f = \frac{dP}{dE} \times E$$

Solving for f we find:

[6] $$f = \frac{\frac{dP}{dE}}{P/E}$$

Referring to the figure below we see that the fraction of labor dedicated to energy production which will maximize the total production (= P×H×[1-f]) is given by the ratio of the slope of the tangent line to the slope of the chord drawn from the origin.



To find the corresponding value of μ/r we substitute the above expression for f into equation [4]. For notational simplicity we set dP/dE = S. We then find:

$$\frac{\mu}{r} = E \times \frac{1 - \frac{(S \times E)}{P}}{\frac{(S \times E)}{P}}$$

This equation can be rewritten as follows:

[7] $$\frac{\mu}{r} = \frac{P}{S} - E$$

Equation [7] can be rewritten as follows:

$$\left(\frac{\mu}{r} + E\right) \times S = P$$

Inspection of the figure makes it clear that

[8] $$\frac{\mu}{r} = E_0$$

To prove that these values of f and μ/r correspond to a maximum of the function P×(1-f) it is necessary to show that the second derivative of this function with respect to f is less than zero. The algebra involved is fairly tedious so I will just state the result:

$$SecondDerivative = \frac{\frac{d^2P}{dE^2}}{f \times (1-f)}$$

when f and μ/r take on the values given by equations [6] and [7] above. Since f and 1-f are both greater than zero and the second derivative of P with respect to E is negative for a curve of the shape shown in the figure, we can conclude that P×(1-f) is a maximum at the values of f and μ/r given above.

Summary

1. The cost of finite non-energy production resources such as labor, land, fresh water, etc must be accounted for in determining the economic benefits of energy production in addition to calculating the energy balance.
2. Energy sources with low net energy balance will tend to have higher non-energy resource costs at a given level of net energy production compared to high net energy balance sources.
3. If the opportunity cost of the non-energy resources required to produce 1 net unit of energy are equal to the value of the unit of energy, then the energy production process has no economic benefit no matter how good energy balance may be.
4. In scaling up energy production to large levels the opportunity cost of using non-energy related production resources can rise dramatically. So if producing large amounts of oil shale involves obtaining 40% of the water rights in some large area of the American west then the opportunity cost of this expansion may be far larger than would be computed from the current market price of water in those areas.
5. Use of cheap abundant energy is limited by marginal utility. That is as we extract more and more energy it becomes more difficult to produce of sufficient economic value to justify the extraction of a marginal unit of energy. This claim implies that a strong motivation has always existed to increase the efficiency with which energy is converted into economic value. Therefore the assertion of some people that as energy becomes more expensive we can easily maintain economic growth via greater efficiency is doubtful.