An Oil Production Model from Roger Bentley

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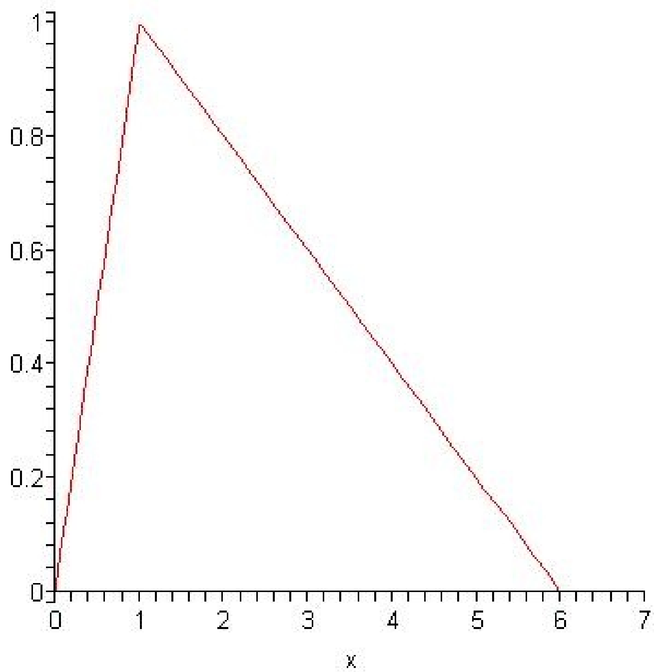
Topic: Supply/Production
Tags: oil production, oil production modeling [list all tags]

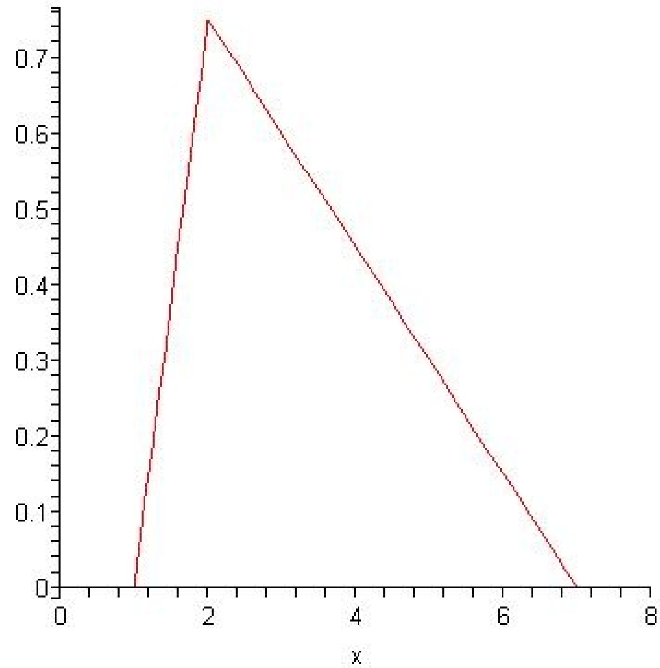
This is a guest article by Dudley Stark, Reader in Mathematics and Probability in the School of Mathematical Sciences, Queen Mary, University of London.

Bentley introduced the following model of oil production on page 204 of Global oil & gas depletion:an overview, and it is dicussed in the book The Last Oil Shock by David Strahan. This posting is meant to explain his model and some results I obtained for it. Consider the following oil production curve:

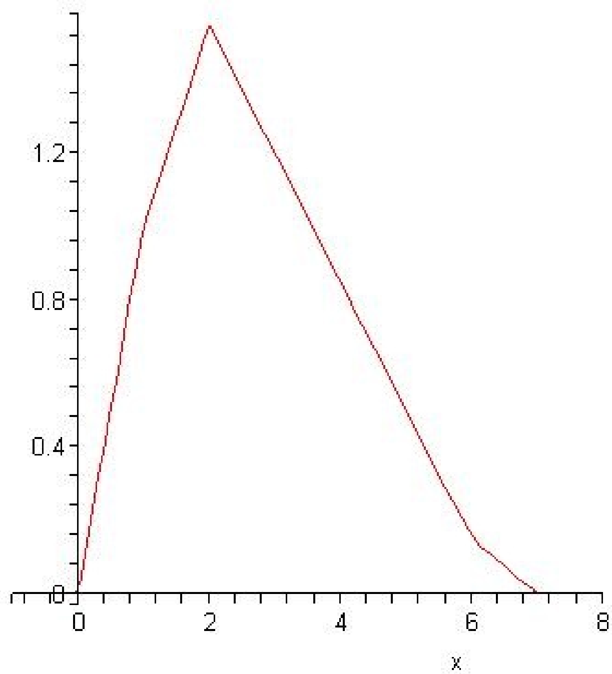


It rises quickly to it's peak at time t=1 and decreases slowly until no oil is produced at time t=6. The idea is that the natural pressure of the oil field causes rapid production initially, after which decline is more gradual. Before and after the peak the curve is linear, so it looks like a triangle.

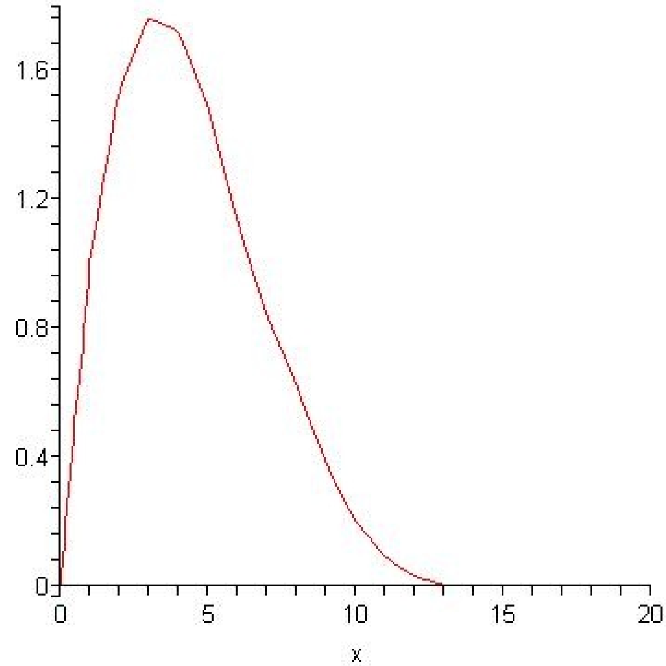
Suppose the next oil field looks the same as the first one, but oil production begins one unit of time later and the total amount of oil produced is only 75% of the oil in the first field. It looks like this:



Adding the production of the two oil fields together gives this production curve:



If you do this eight times, each time shifting the start of production by one time unit from the previous oil field and also making the amount of oil produced 75% of the previous oil field, you get a curve like this:



It is starting to look like a plausible oil production curve. Note, however, that it is not too realistic because, for one thing, the curve is linear in between integers.

In my paper Peak Production in an Oil Depletion Model with Triangle Field Profiles, accepted to appear in *Journal of Interdisciplinary Mathematics*, I analyze Bentley's model. The production curve of the *i*th oil field is assumed to look like this:

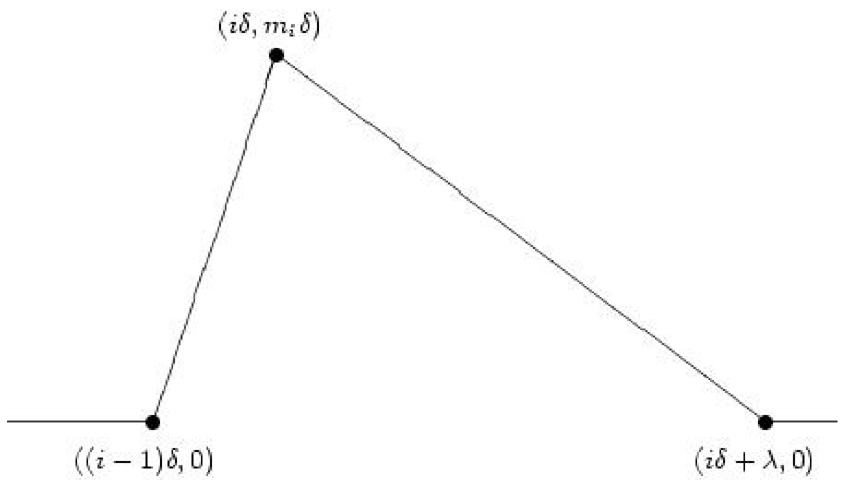
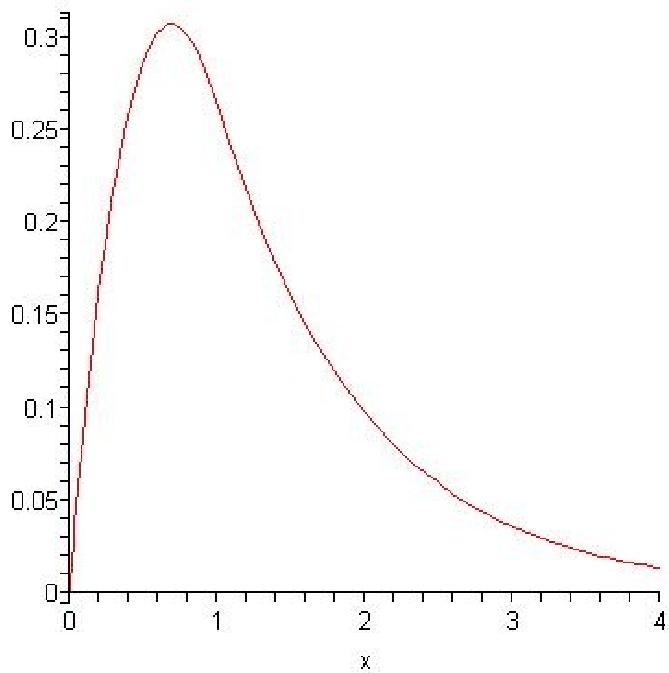


Figure 1: Profile $g_i(t)$ of the ith oil field.

Note that the amount of time from the start of production until peak is delta for each curve and the amount of time from peak to the end of production is lambda. We assume that delta is smaller than lambda. The *i*th field begins production at time (i-1)delta. The amount of production at peak is determined by a parameter m_i which also determines the total amount of oil produced by the *i*th field.

I show that in this model, assuming that the m_i are decreasing, the resulting total oil production curve is concave on the interval [0,lambda] and is decreasing on the interval [lambda,infinity). The oil production curve therefore takes it's maximum on the interval [0,lambda]. If the m_i decrease geometrically, then in addition the oil production curve is convex on the interval [lambda,infinity).

I also show that if delta=alpha/n for a constant alpha and a parameter n and if the m_i=f(i/n) for a decreasing function f, then as n goes to infinity the oil production curve converges to a smooth (meaning differentiable) curve which is again concave on [0,lambda] and decreasing on [lambda,infinity). Here is an example of a smooth curve:



These curves are concave on the interval [0,lambda] and attain their unique peaks there. If f is a negative exponential function (which is the continuous analogue of a decreasing geoometric), then the oil production curves are convex on [lambda,infinity). I show that for these limiting curves it is not possible to have 1/2 of the total oil produced at the time of peak production and in fact they have at most about 35% of the total oil produced at the time of peak production.

A related paper is "Oil Production: A probabilistic model of the Hubbert curve" by Bertrand Michel, who spoke at the ASPO-V conference, to appear in Mathematical Geosciences. In his model the field sizes are chosen uniformly at random from a truncated Pareto distribution. Conditional on the size of the field, the starting time for oil production is Gamma distributed with parameters depending on the size. The field profile shapes are constant; in his application they are given by splines. So Michel's model is similar to Bentley's but harder to analyze and probably more realistic. He uses it to model North Sea oil production (see Figure 11 on page 18). The fit is pretty good except for a decrease in production caused by the "Piper disaster", which couldn't be anticipated by the model.

I perform a similar analysis for Bentley's model of gas production, in which the *i*th field production profile has a trapezoidal shape:

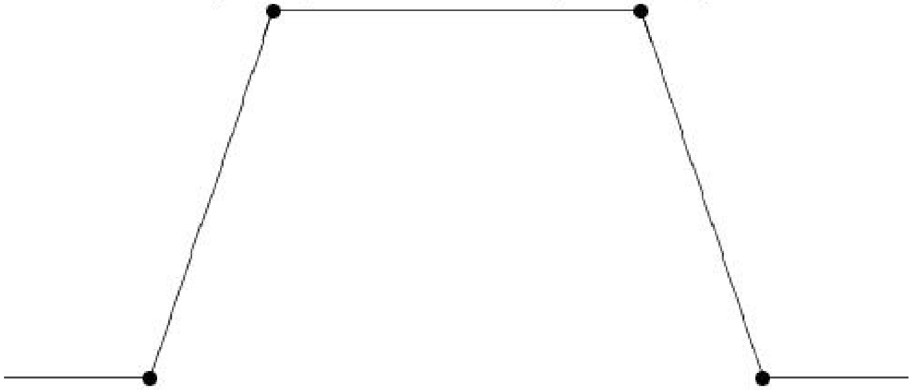


Figure 2: Profile $h_i(t)$ of the ith natural gas field.

The trapezoidal shape is suppposed to be more realistic for gas profiles because gas production is constrained by the pipes which transport it.

Conclusion

Bentley's model generates plausible looking asymmetric production curves in which less than 50% of the total oil has been produced at the time of peak production. The fact that they are asymmetric is not surprising: see the paper "Testing Hubbert " in which it is found that the asymmetric exponential curve is most often the best fit when six different types of curves are fitted to real data.

I'd also like to point out that, though I think papers like this one are important because oil production is important, no one else seems to be doing this kind research in the mathematics of oil production and therefore many math journals wouldn't consider publishing a paper like this. The reason is that they only want to publish papers that reference other published papers in respected journals and this paper only references oil industry papers and peak oil books, as well as another paper written by myself. I was pleased to learn about Michel's paper, which is published in Mathematical Geology, but references a lot of oil industry papers and is like an applied statistics paper.

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Previous post from Dudley:
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