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A Different Way to Perform the Hubbert Linearization

Posted by Sam Foucher on August 18, 2006 - 11:34am
Topic: Supply/Production
Tags: hubbert linearization, logistic, urr [list all tags]

A quick post about a different manipulation of the logistic differential equation. By using the first derivative, we get a new way to perform the Hubbert linearization. Some results are given on Norway and the US oil production.

[Updated by Khebab on 08/18/2006 at 02:36 PM EDT] After some thinking, I came up with a simple way to combine the two linearizations (see text below).

The logistic differential equation relates the production P to the cumulative production Q as following:

$$P=dQ/dt=KQ(1-Q/URR)$$

This equation is the basis for the standard Hubbert Linearization (HL) technique as explained by Stuart here. Let's differentiate P:

$$dP/dt=(dP/dQ)(dQ/dt)=(dP/dQ)P=K(1-2Q/URR)P$$

We get the following new equation for the production relative annual increase:

$$(dP/dt)/P=K(1-2Q/URR)$$

Therefore, for a logistic curve, the relative annual production increase is a linear function of the cumulative production. The logistic parameter (K and URR) can be estimated by a simple linear fit using the annual production increase versus cumulative production representation (see Fig. 2 below). The new HL line crosses zero at half the URR value. There are different ways to numerically estimate the first derivative, for instance we can use $dP/dt=P(t)-P(t-1)$ or $dP/dt=(P(t+1)-P(t-1))/2$. The second approach is probably more accurate because less sensitive to noise and it is not shifted as the first one. We can compared the results of the standard HL and the second HL techniques on the Norway production (Fig. 1 and Fig. 2) and the US production (Fig. 3 and Fig. 4).

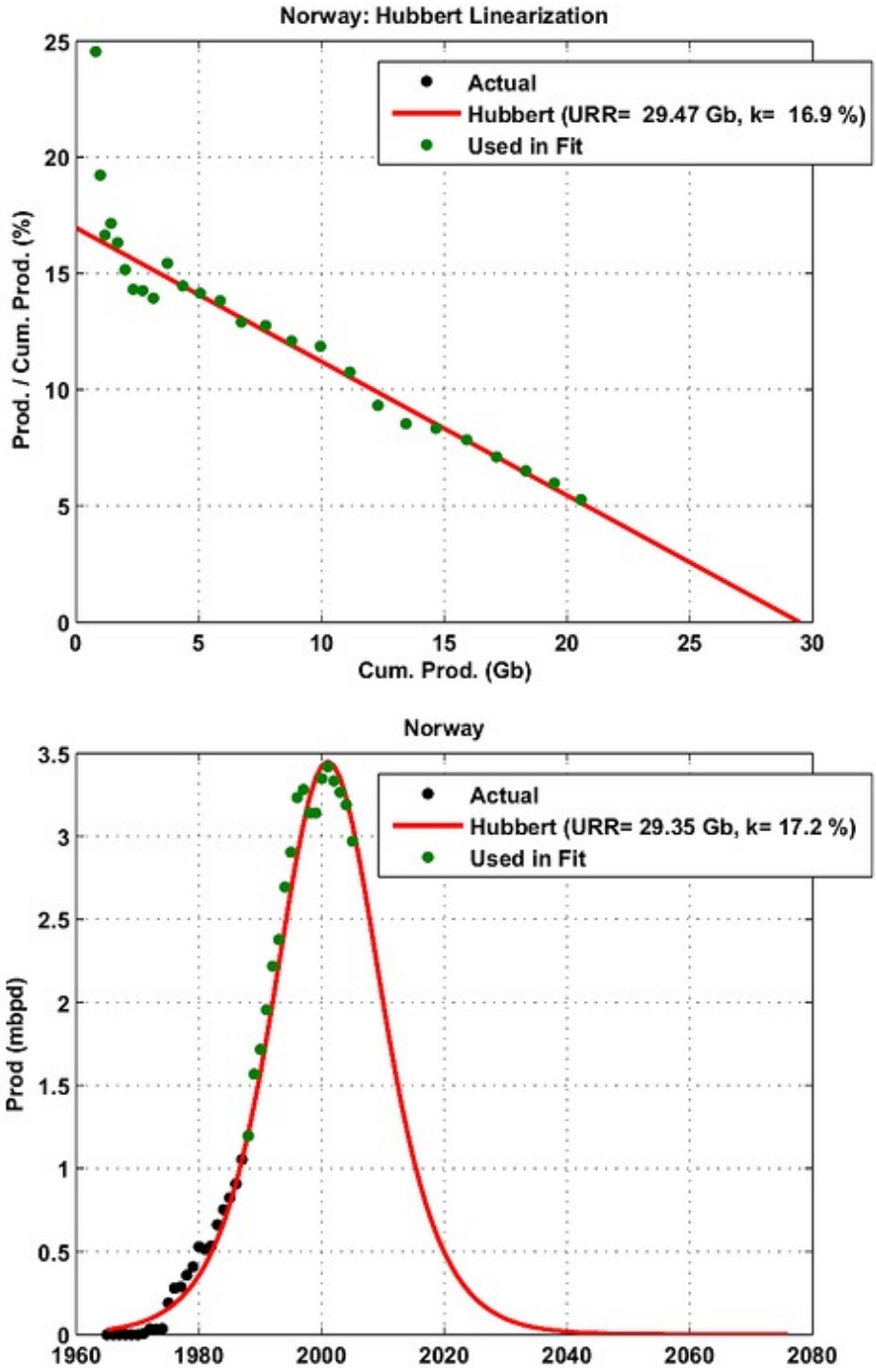

Fig. 1- Standard Hubbert linearization (top) and resulting logistic curve (bottom). The peak date is determined by matching cumulative productions for the last year. (data from BP 2006, all liquids excluding refining gains). Click To Enlarge.

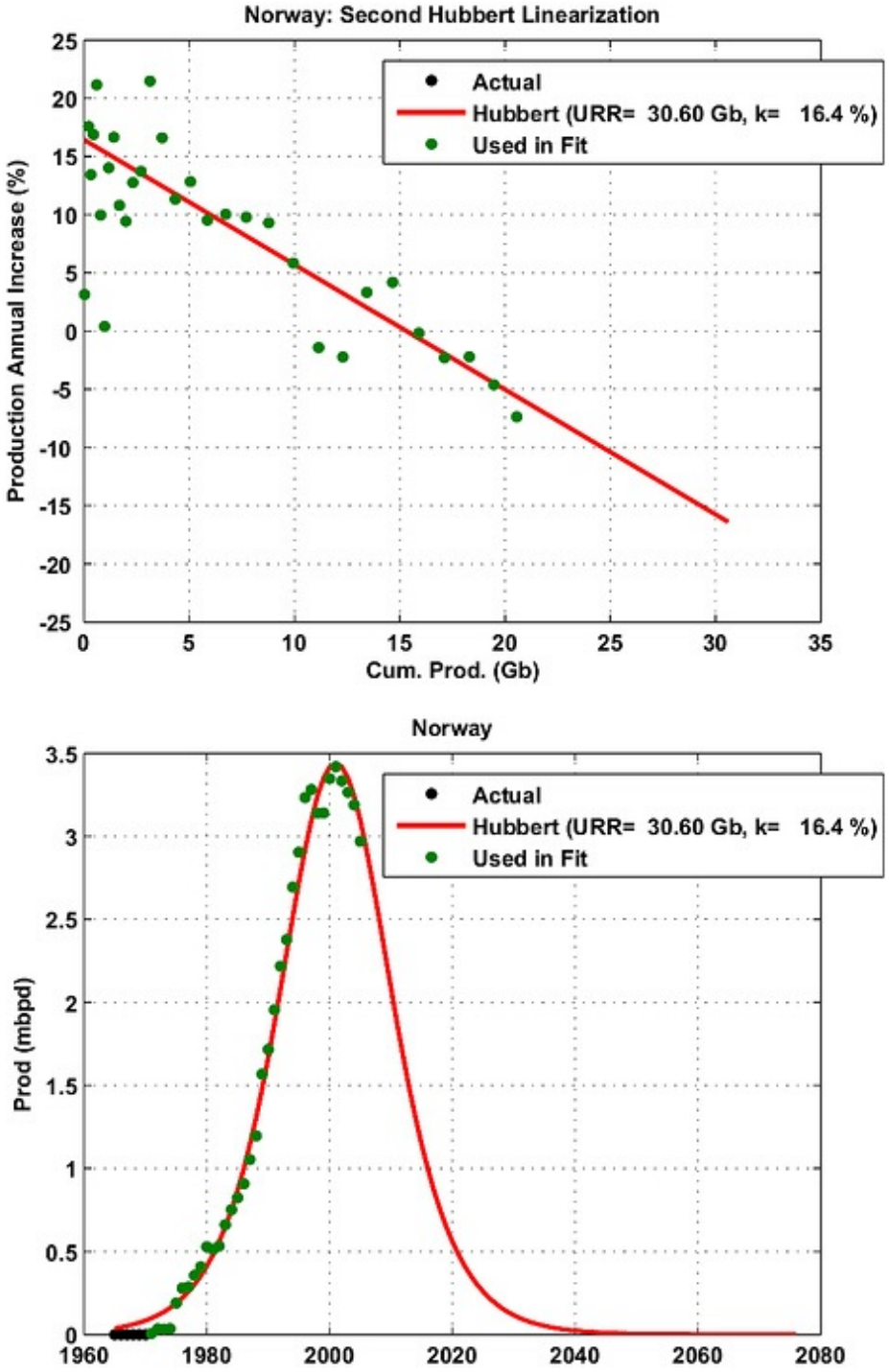

Fig. 2- Hubbert linearization on the production annual increase (top) and resulting logistic curve (bottom). The peak date is determined by matching cumulative productions for the last year. (data from BP 2006, all liquids excluding refining gains). Click To Enlarge.

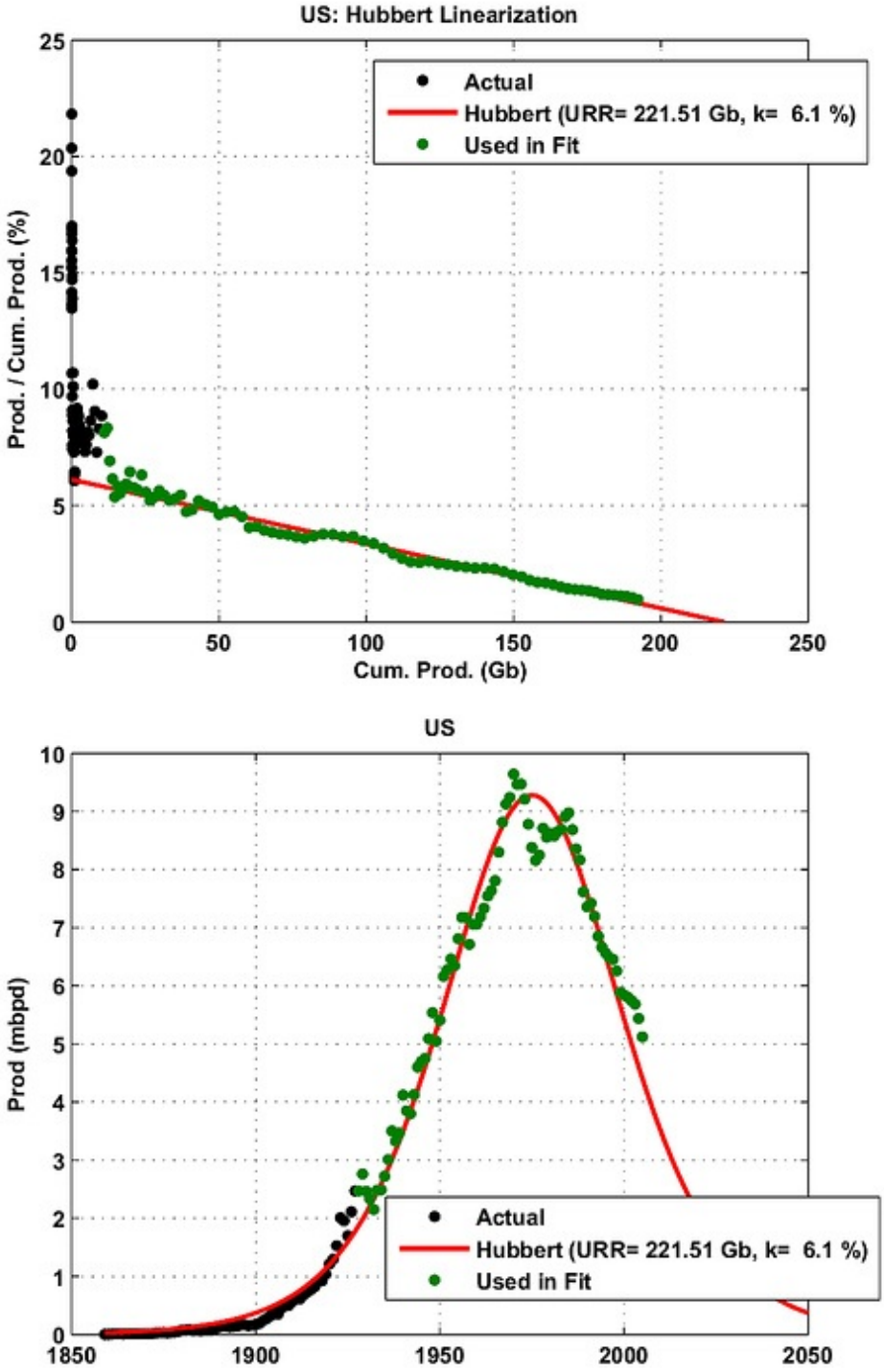

Fig. 3- Standard Hubbert linearization (top) and resulting logistic curve (bottom) for the US. The peak date is determined by matching cumulative productions for the last year. (data from the EIA, crude oil only). Click To Enlarge.

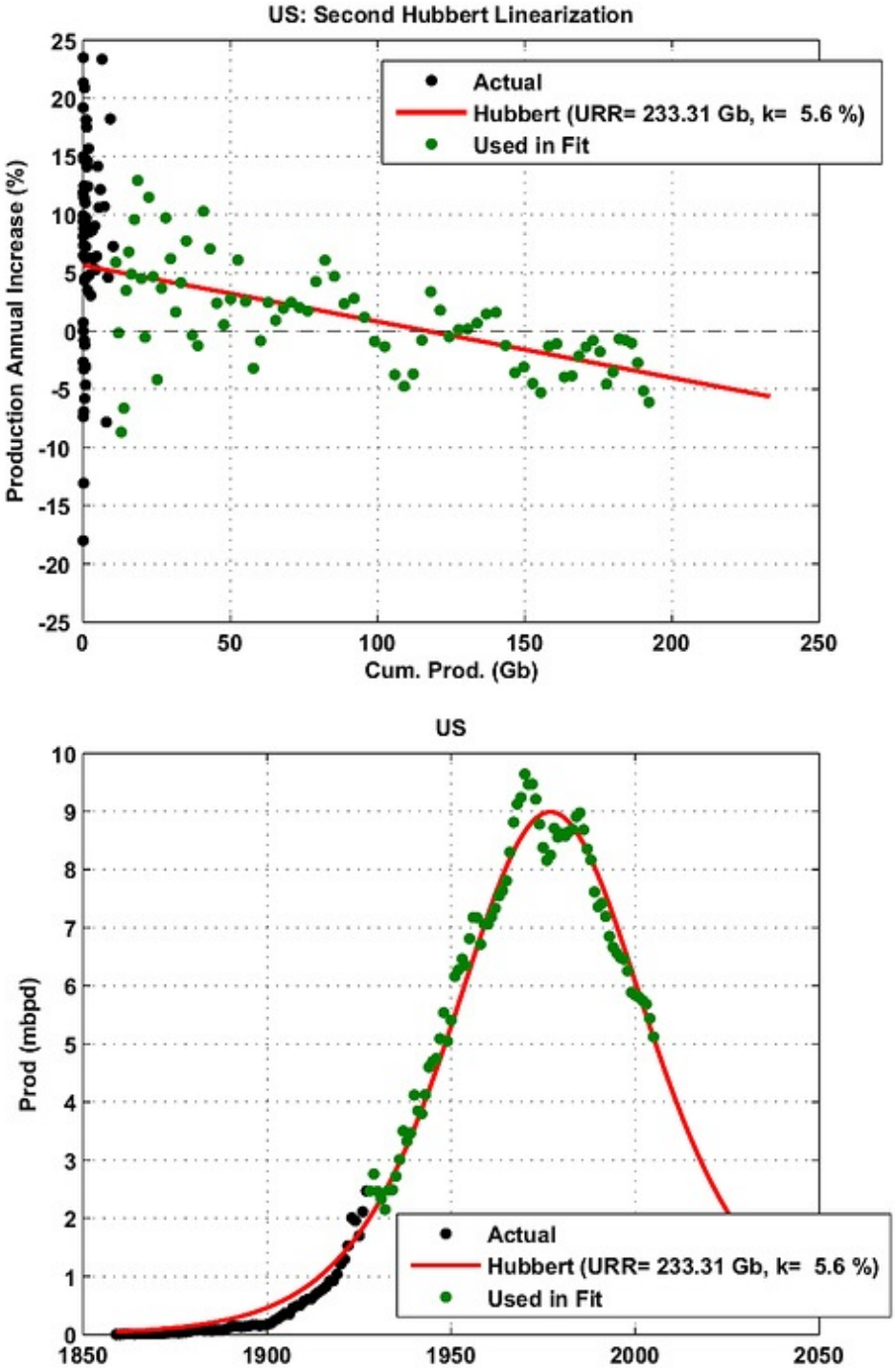

Fig. 4- Hubbert linearization on the production annual increase (top) and resulting logistic curve (bottom) for the US. The peak date is determined by matching cumulative productions for the last year. (data from the EIA, crude oil only). Click To Enlarge.

A few comments:

1. this approach is much more sensitive to noise because of the use of the production first derivative and seems to give more reliable estimates for the URR than for K.
2. the data representation is more symmetric compare to the standard HL approach which is very sensitive to noise for low cumulative production values (therefore, early production

data points are generally excluded).

3. the two HL techniques could be combined.

[Updated by Khebab on 08/18/2006 at 02:36 PM EDT] After some thinking, I came up with a simple way to combine the two linearizations: if you look at the original HL equation and the proposed second HL, they only differ by a factor 2 on the slopes, their intercept with the y-axis being the same and equal to K. Therefore, the two representations could be mixed together by multiplying the cumulative production by a factor two in the second representation (shown as blue dots on Fig. 5).

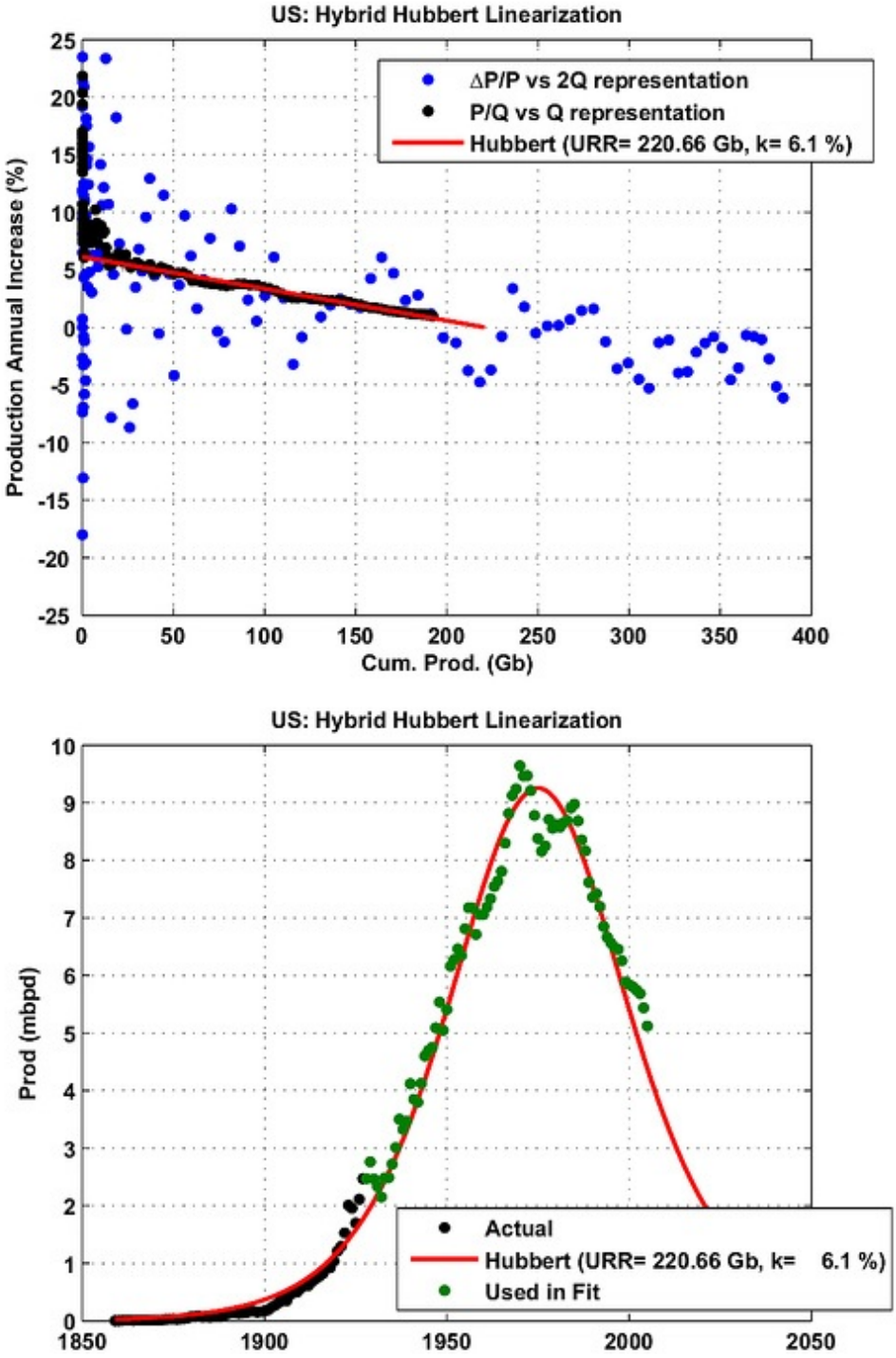

Fig. 5- Hybrid Hubbert linearization combining the production annual increase (blue points)

and the standard P/Q representation (black points), the resulting logistic curve (bottom) for the US. The peak date is determined by matching cumulative productions for the last year. (data from the EIA, crude oil only). Click To Enlarge.

Other stories on TOD about the Hubbert Linearization here. The Datasets used in this post can be found here.