The Oil Drum
DISCUSSIONS ABOUT ENERGY AND OUR FUTURE

Norway and the Parabolic Fractal Law

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Topic: Geology/Exploration
Tags: m. king hubbert, norway, parabolic fractal law, urr [list all tags]

Norway can be considered as the poster child of the Hubbert curve modeling approach with a production profile that is remarkably close to the logistic curve. Last time, we attempted to apply the *Parabolic Fractal Law* (noted PFL) to Saudi Arabia. Despite using very partial data, the PFL seemed to point toward an *Ulimate Recoverable Ressource* (URR) around 250 Gb when the PFL curvature is set to the value *-0.07* established by Jean Laherrère for the entire world. In the present post, we propose to apply the same approach on Norway's oil field size distribution. The results seem to confirm that the PFL with a curvature value around -0.07 could be a good predictor of the URR.

Notations:

URR= *Ultimate Recoverable Ressource*
PFL= *Parabolic Fractal Law*
HL= *Hubbert Linearization*
Mb= *Millions of barrels*
Gb= *Billions of barrels*

Norway's oil production is one of the most well documented production with complete data freely available on the Internet [10]. Production has been booming in the late 80s with a 50% growth in production between 1987 and 1992. However, production has peaked in 2001 and has declined dramatically afterward with a decline rate that mirrors the previous strong growth.

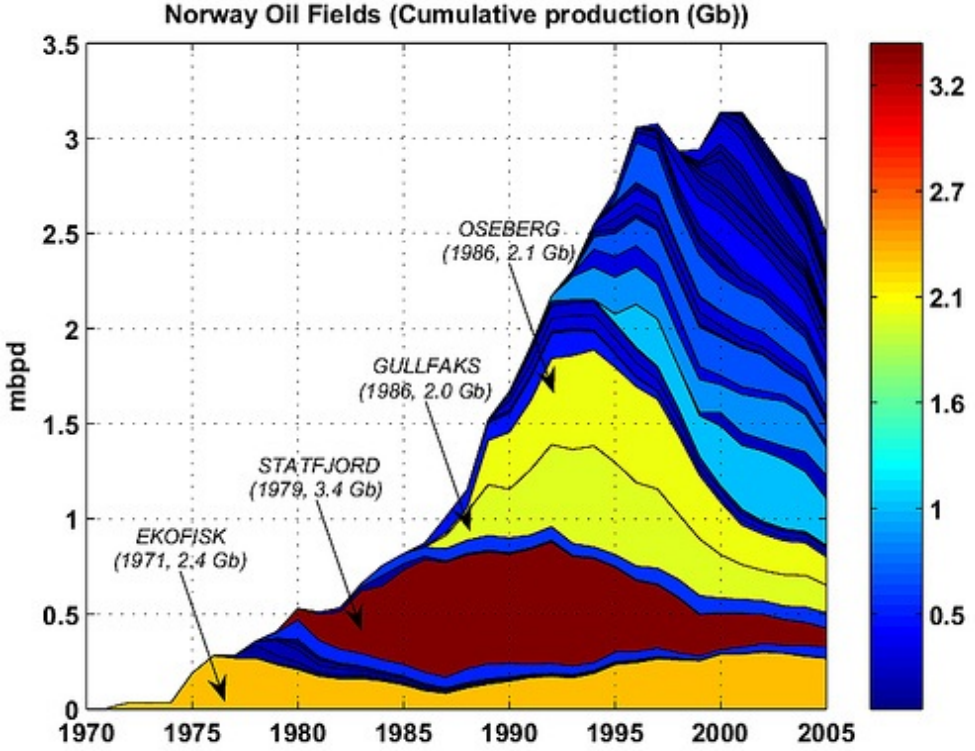
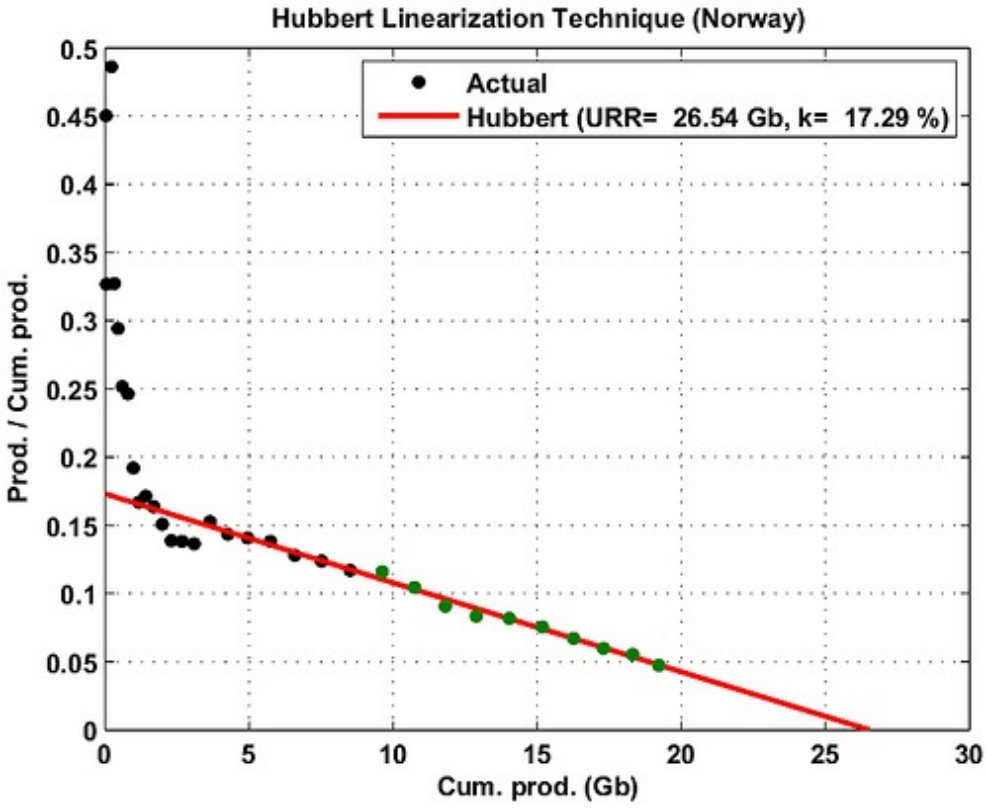


Fig. 1- Norway oilfields (crude oil only). GraphOilogy.

As we can see on Fig. 2, the Hubbert approach gives rather good results. The estimated URR is 26.54 Gb (crude oil only, no condensates) and the observed cumulative production is 19.2 Gb (72.5% of the URR).



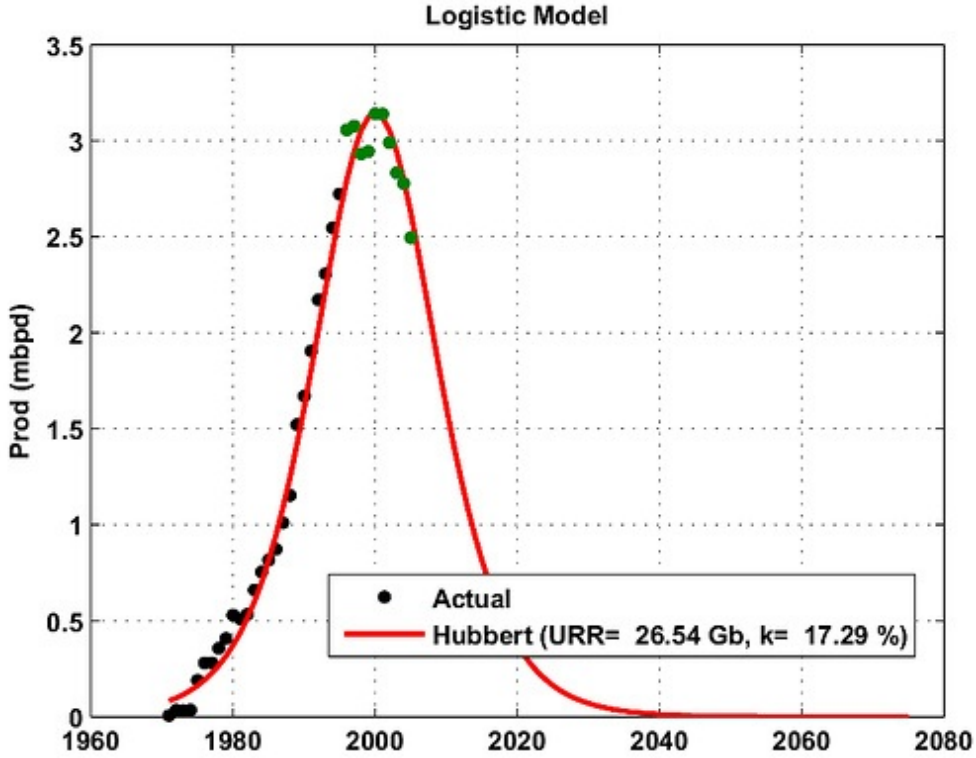


Fig. 2- Hubbert Linearization technique applied on Norway Production (green points are the data used for the fit).

We can appreciate how well the curve fitting is working for Norway by comparing with some reserve estimates given in Table I.

Source	Year	Past Production (Gb)	Reserve (Gb)	URR (Gb)
ASPO [7]	End 2001	15.2	16	31.2
BP [9]	end 2005	19.2	9.7	28.9
Oil & Gas Journal [8]	end 2005	19.2	7.7	26.9
World Oil [8]	end 2004	14.0	9.86	25.0

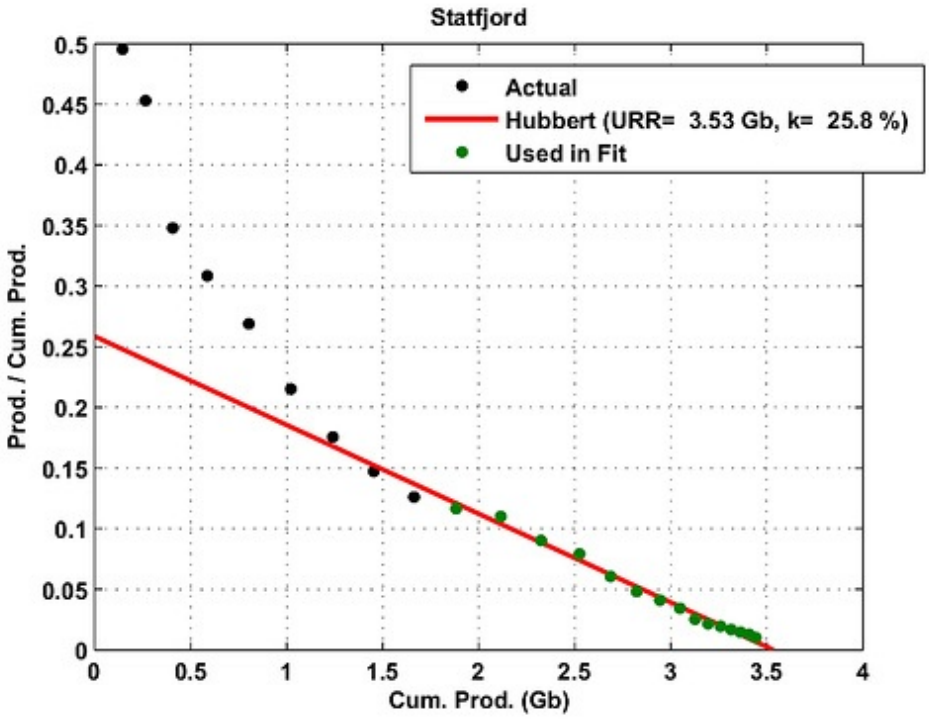
Table I. Some URR estimates from various sources. Past production is for crude oil only. Reserve estimates may include NGL.

The logistic growth rate (or decline rate) *K* for Norway is very large at 17% . One reason for that steep production growth and decline is maybe due to the very large decline rates observed on the top 5 fields as shown on Table II. Four of the top fields have values for *K* ranging from 25% to 37%! Fig. 3 gives the result of the logistic curve modeling on *Statfjord* which is at 98% of its URR. Only *Ekofisk* (Fig. 4), one of oldest field, can be seen as the remaining backbone of Norway's

production with only a logistic decline rate at 8% and only 47% of its URR produced.

Field	Q(2005) (Gb)	Starting Year	URR (Gb)	K (%)
Statfjord	3.45	1979	3.53	25.8
Ekofisk	2.40	1971	5.10	8.3
Oseberg	2.12	1986	2.21	32.7
Gullfaks	2.04	1986	2.11	31.6
Troll	1.08	1990	1.43	36.8

Table II. Norway Top 5 fields: cumulative production in 2005, first year of production, URR and logistic growth (K) estimates using the HL technique.



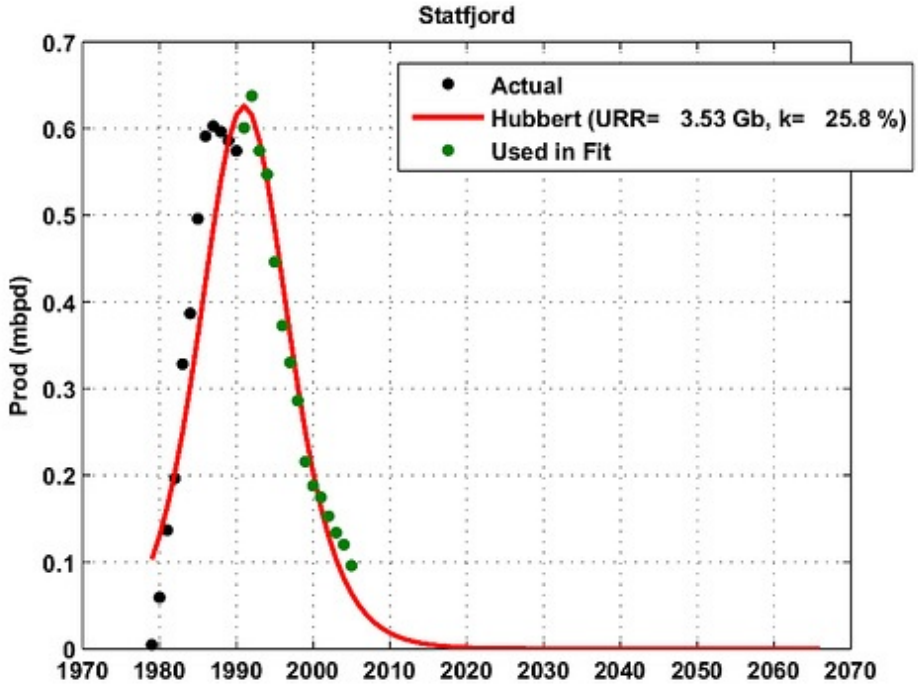
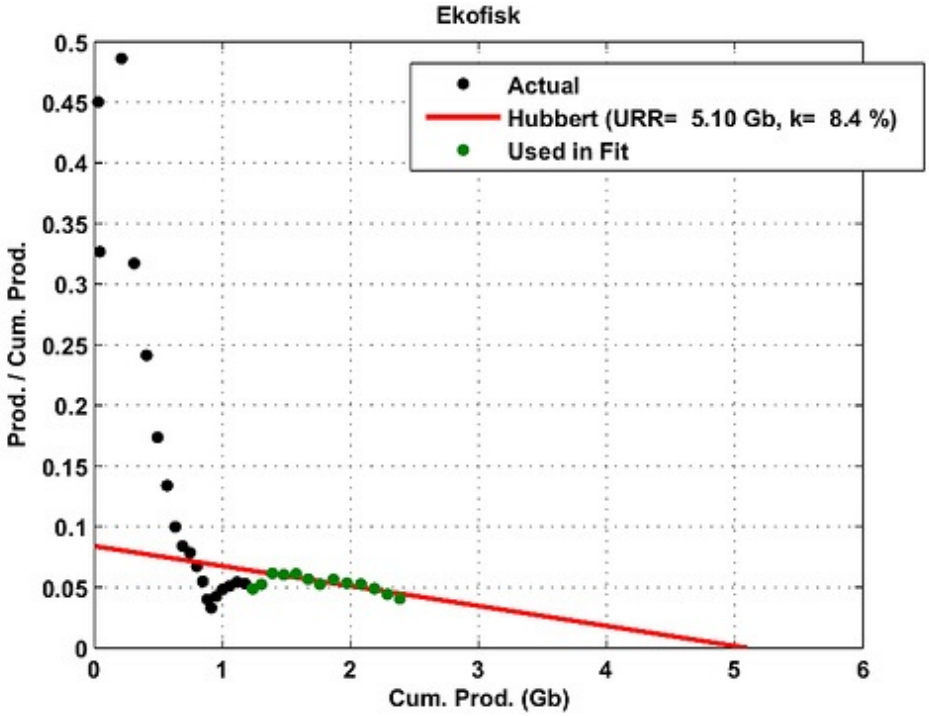


Fig. 3- HL technique applied on the Statfjord oilfield. The logistic curve peak position is computed
in order to have the same cumulative production in 2005.



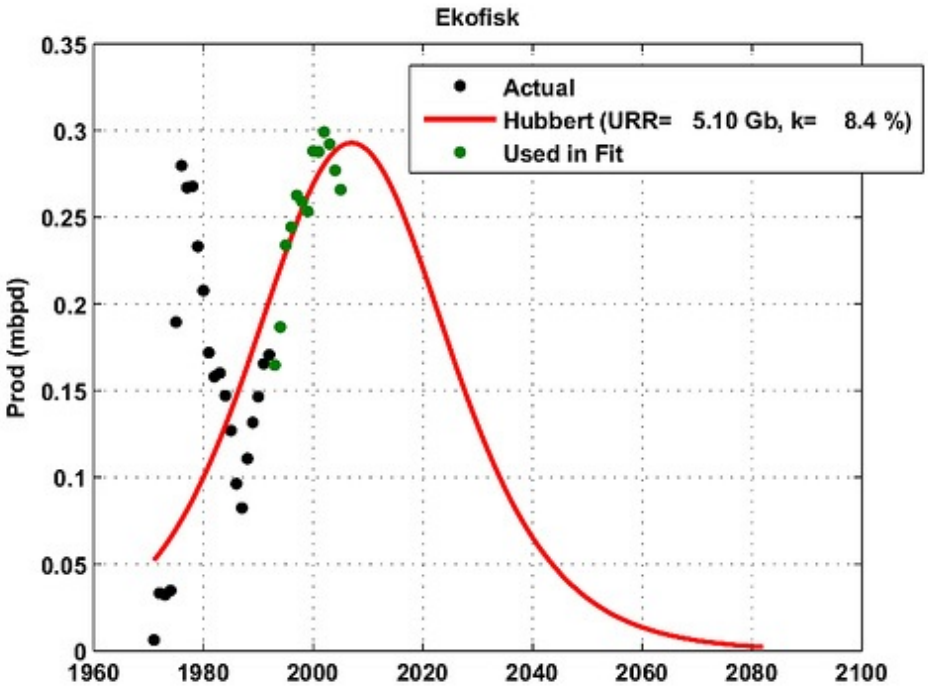


Fig. 4- HL technique applied on the Ekofisk oilfield. The logistic curve peak position is computed in order to have the same cumulative production in 2005

The data for Norway contains production profiles for 47 oilfields. In order to get a more precise estimate of the PFL parameters, we perform the HL technique on each of the top 16 fields represented as empty circles on Fig. 5. Note that this refinement step does not change much the top of the PFL curve which is already very mature.

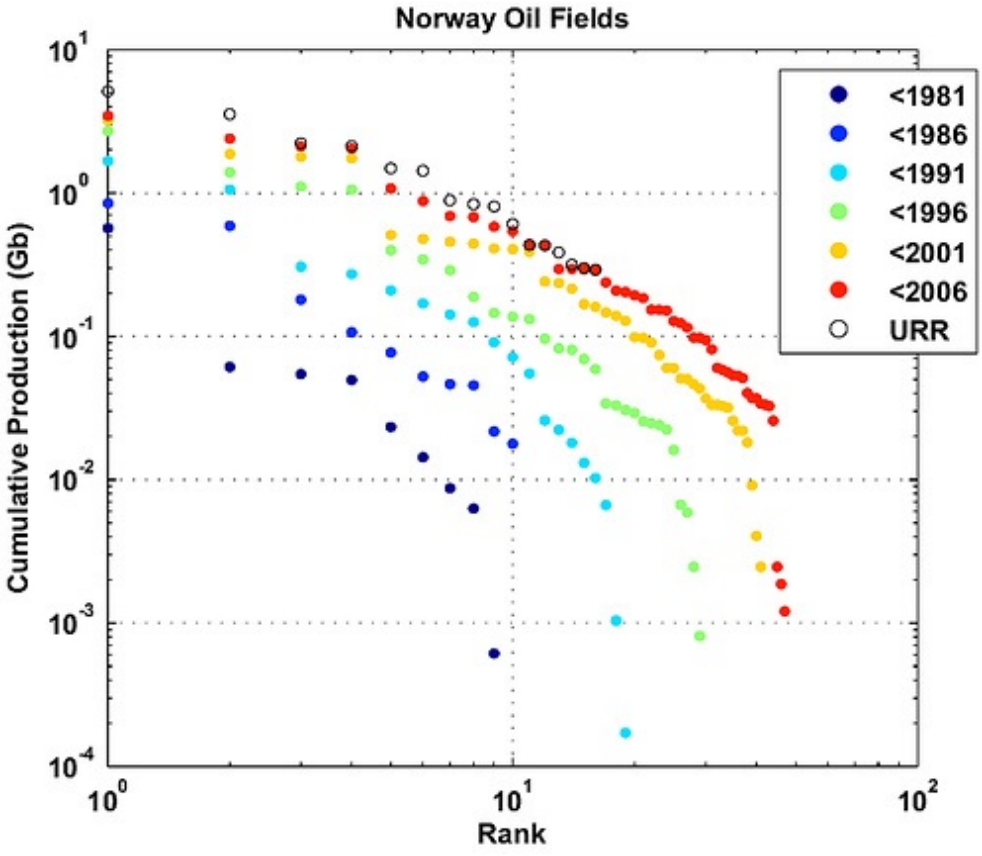


Fig. 5- Field cumulative production values displayed in a log(size)-log(rank) plane at different points in time.
The empty circles are the URR estimates for the 16 top fields using the HL technique.

The next step is to try to fit a quadratic polynome using the data representation shown on Fig. 6. Only the top 16 fields are used for the fit. The resulting curvature is surprinsingly low at *-0.31* which is more than four times the value obtain for the UK and the world (*-0.07*) [2]. I don't have a definitive answer on why this value is so low, one possibility is the dataset contains too few oil fields (only 47). One clue: In [5] page 26, Jean Laherrère reports the following:

> *In UK only 52% of the discoveries are developed representing 89% of the reserves. In Norway, only 33% of the discoveries are developed representing 83% of the reserves.*

Table III is from the same document and gives some details on the distribution of reserves for the UK and Norway (NW).

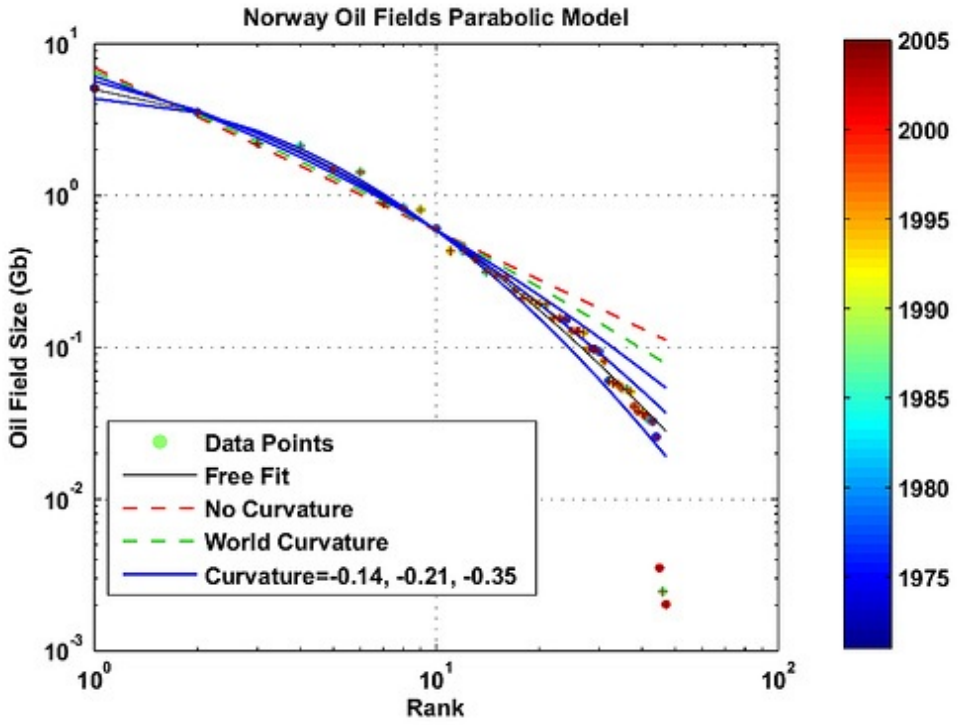


Fig. 6- Estimation of various PF Laws with different fixed curvature values.
Each data point is color coded according to the oil field age..

Status 2001	UK	UK	UK	UK	NW	NW	NW	NW
	O+C Gb	nb	% Gb	% nb	O+C Gb	nb	% Gb	% nb
developed	31,7	276	89	52	26,3	64	83	33
undeveloped	3,9	254	11	48	5,3	128	17	67
Total	35,6	530	100	100	31,6	192	100	100

Table III. from Jean Laherrère [5], page 26: O= crude oil, C= Condensate, nb= number of fields.

Now, let's assume that we have a total number of fields around 200. This number will intersect the different Parabolic Fractal curves and is represented as an orange dotted line on Fig. 7. The intersection of this line with the different curves gives different URR and minimum size values as shown on Fig. 8. We can see that the world curvature (the orange disc) leads to an URR close to

the ASPO and the official reserve numbers (the minimum field size is then around 7 Mb).

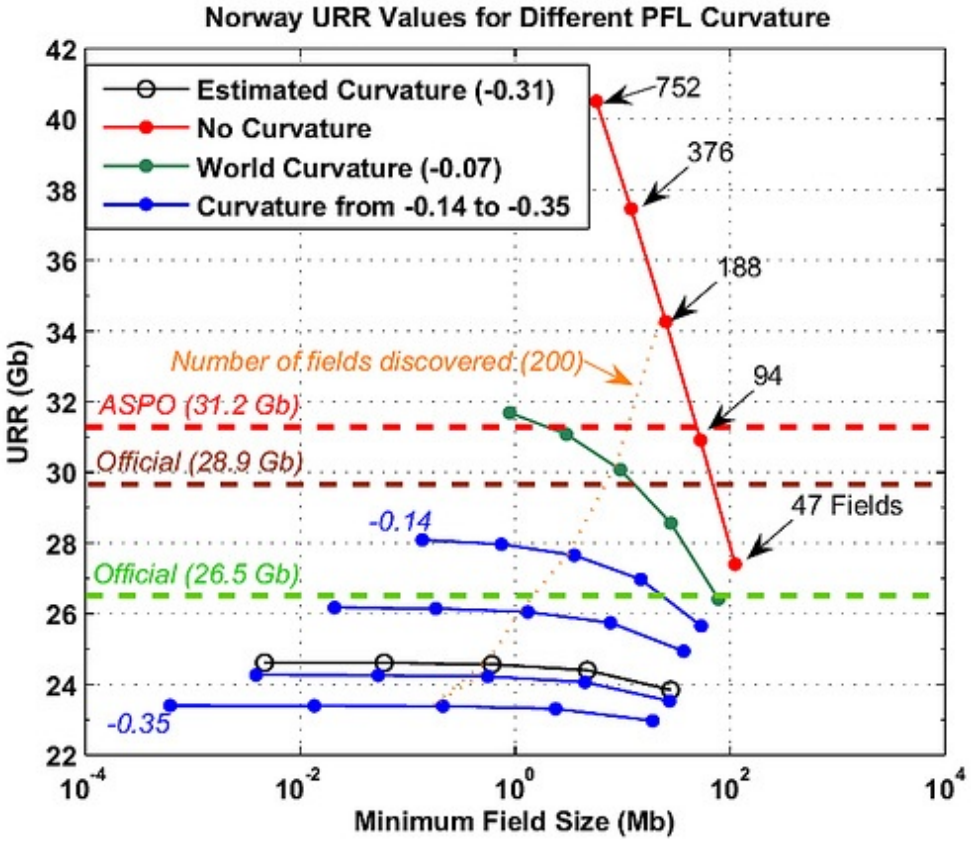


Fig. 7- Derived URR from the PFL shown on Fig 6 for different curvature values and number of fields.
The orange dotted line is the isoline corresponding to 200 fields.

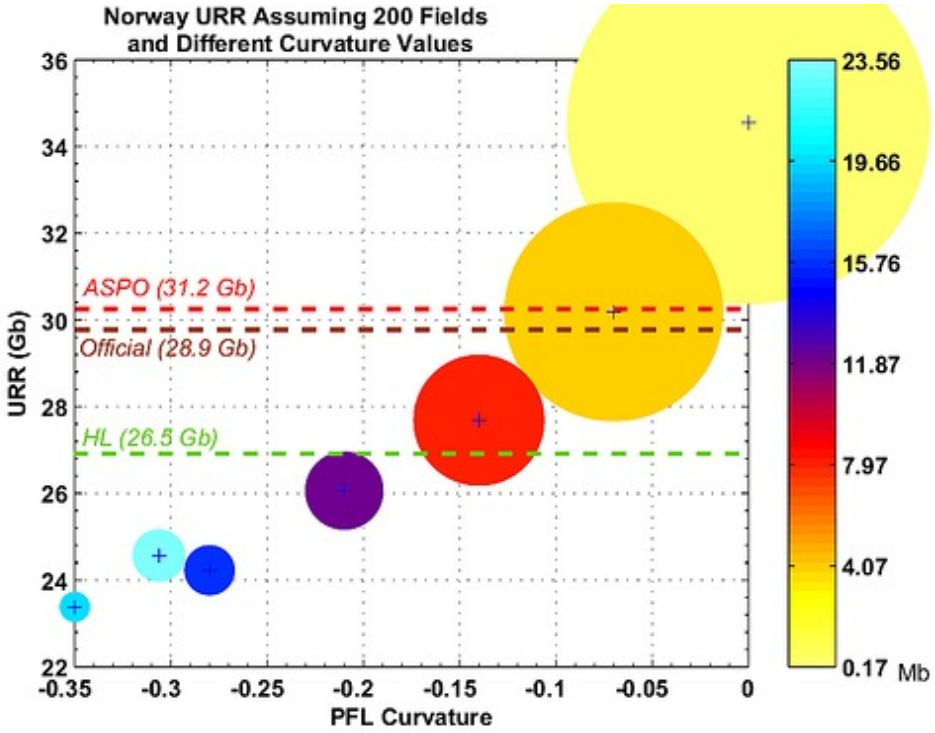


Fig. 8- Derived URR from Fig. 7 by fixing the number of fields to 200 (dotted orange line on Fig.

8).

The disc size and color is function of the minimum field size.
HL is the URR estimate from the Hubbert Linearization shown on Fig. 2.

Conclusion

In summary, we have applied the Parabolic Fractal Law to the Norway dataset (crude oil only) composed of 47 oil fields. The individual URR for the top 16 fields are estimated using the Hubbert Linearization technique. Because production from large fields is very mature, this last step will not affect significantly the PFL parameters. Different PFL are then fitted in the log(size)-log(rank) representation. The free fit with no constraints on the parameters leads to a very low curvature (*-0.31*). One possible explanation is that the dataset is incomplete and contains only a fraction of the oil fields that have been discovered (~33%). However, the PFL with a curvature of *-0.07* gives a credible URR range (26-32 Gb) covering most of the published URR estimates. Consequently, we have been able to put together different pieces of information about Norway's production:

1. the Hubbert linearization gives an URR at 26.5 Gb
2. the number of fields discovered is around 200
3. the PFL + world curvature gives an URR between 26 and 32 Gb
4. the PFL + world curvature + 200 fields gives an URR at 31 Gb

References

[1] TOD: An Attempt to Apply The Parabolic Fractal Law to Saudi Arabia
[2] GraphOilogy: What Can We Learn From The Oil Field Size Distribution?
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[4] Jean Laherrère: “Parabolic fractal” distributions in Nature. (in French)
[5] Jean Laherrère: Estimates of Oil Reserves
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[8] World Proved Reserves of Oil and Natural Gas, Most Recent Estimates
[9] BP: Statistical Review of World Energy 2006
[10] Norwegian Petroleum Directorate