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An Attempt to Apply The Parabolic Fractal Law to Saudi Arabia

Posted by Sam Foucher on June 17, 2006 - 9:50am
Topic: Geology/Exploration
Tags: parabolic fractal law, saudi arabia, urr [list all tags]

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Many natural object geometry are well described by a fractal (e.g. a coastline). In particular, fractal self-similarity is a powerful concept that has been investigated by Benoît Mandelbrot. However, in practice the self similarity law is not always perfectly respected. To remedy to this, Jean Laherrère has proposed the Parabolic Fractal Law (PFL) which adds a parabolic deviation to the pure self-similar law, I quote: *A complete or near complete distribution of the larger objects, which in practice are usually readily identified and quantified, can be used to define the parabola following a rule of self-similarity, and hence describe the full distribution down to the smallest object. The distribution can in turn be used to determine the total population of the objects*. I believe that the PFL could be a complementary tool to analyze production data under a different angle especially when the Hubbert Linearization technique does not produce a clear result. For instance, when applied to the United Kingdom production data, the resulting Ultimate Recoverable Ressource (URR) is very close to the value estimated by the Hubbert Linearization technique. I intend here to apply this technique to Saudi Arabia oil fields. Despite using coarse oilfield size estimates, the PFL seems to point toward an URR close to the ASPO estimate at 270 Gb.

Why self-similarity?

The distribution of oil field sizes is characterized by a few large fields (the king, the queens, etc.) at one end and a large pool of small fields at the other end. This pattern can be closely represented by a self-similar process. To understand this, let's imagine that the biggest field has a size of 2,024 Mb. The next fields in size will be 2 fields of size 1,024 Mb. Consequently, the self-similarity rule is to multiply by two the number of fields at each stage and divide by two their sizes. A convenient way to reveal the self-similarity law is to display the log of the ranked field sizes versus the log-rank as shown on Fig. 1. If the process is perfectly self-similar the points are distributed along a straight line with *-1* slope value.

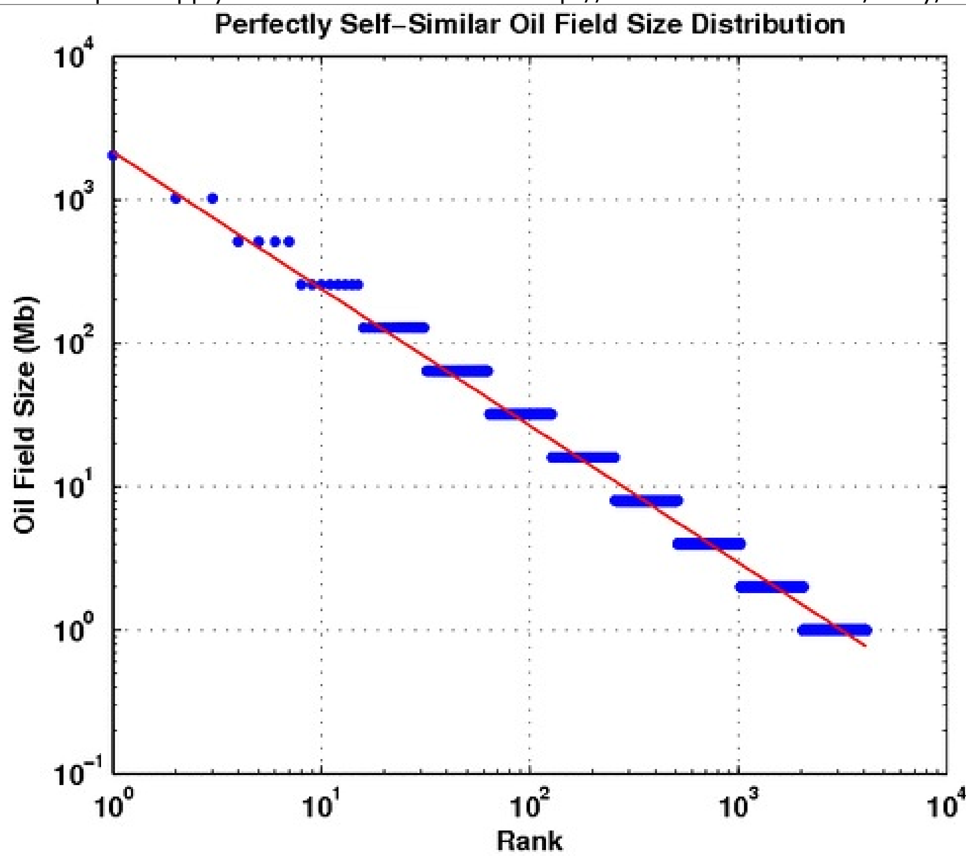

Fig. 1 Example of a simple self-similar object size distribution represented in a log(rank)-log(size) plane. The red line has a slope equals to -1.

The Parabolic Fractal Law (PFL) is an unperfect self-similar law where a quadratic term is added:

$$\log(Size(i)) = a + b.\log(i) + c.\log(i)^2$$

where *Size(i)* is the size of the oil field of rank *i*. we call *c* the PFL curvature. In case of perfect self-similarity, we have $c=0$ and $b=-1$. Jean Lahèrrere has estimated the curvature for the world (excluding North America) and came up with the value $c=-0.1518/\log(10)=-0.07$. Interestingly, we get also the same value for the UK oil fields (see GraphOilogy for details). Once the PFL parameters are estimated we can derive an URR value by computing the area under the PFL curve given a small field size cutoff:

$$URR = Sum\_i(a + b.\log(i) + c.\log(i)^2)$$

$$\text{with: } a + b.\log(i) + c.\log(i)^2 > Size\_Min$$

Application to Saudi Arabia

Unfortunately, there is no public database on Saudi Arabia oilfields. We don't need to get an exhaustive dataset but only a few estimates about the size of the largest fields. I found some data about the top 9 fields from various sources on the web and from Simmons's book (Twilight in the Desert).

Field	URR (Gb)	Discovery Date
Ghawar	66-100	1938
Safaniya	21-36	1951
Shaybah	18-18	1969
Manifah	17-17	1957
Berri	10-25	1965
Abqaiq	10-15	1941
Zuluf	12.0-14.0	1965
Qatif	8.4	1965
Abu Safah	6.0	1969

Table I. Size estimates for Saudi Arabia Top 9 fields (src: IHS, Simmons).

Because we have so little data, it will be difficult to reliably estimate a valid parabolic curvature. So we proceed as following:

1. The Field URR are ranked according to their size and represented in a log(rank)-log(URR) plane as shown on Fig. 2. When only an interval is available, we take the center value.
2. a robust linear fit (no curvature) is applied on the data points (red line on Fig. 2).
3. A parabolic model is then fitted using the slope established in 2) as first guess for the linear term *b* and a fixed curvature value *c* (blue lines on Fig. 2). The algorithm used for this step is the Levenberg-Marquardt algorithm.
4. The URR values are estimated from the areas under the PFL curves conditionally to a particular minimum oil field size as shown on Fig. 3.

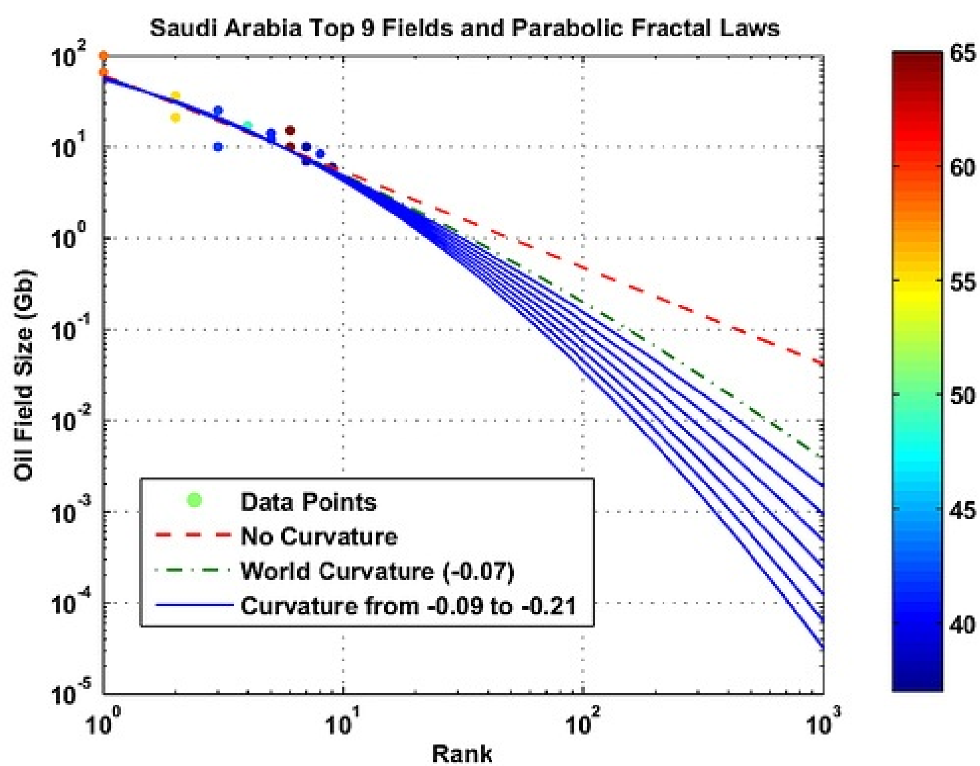

Fig. 2. Estimation of various PF Laws with different fixed curvature values. Each data point is color coded according to the oil field age.

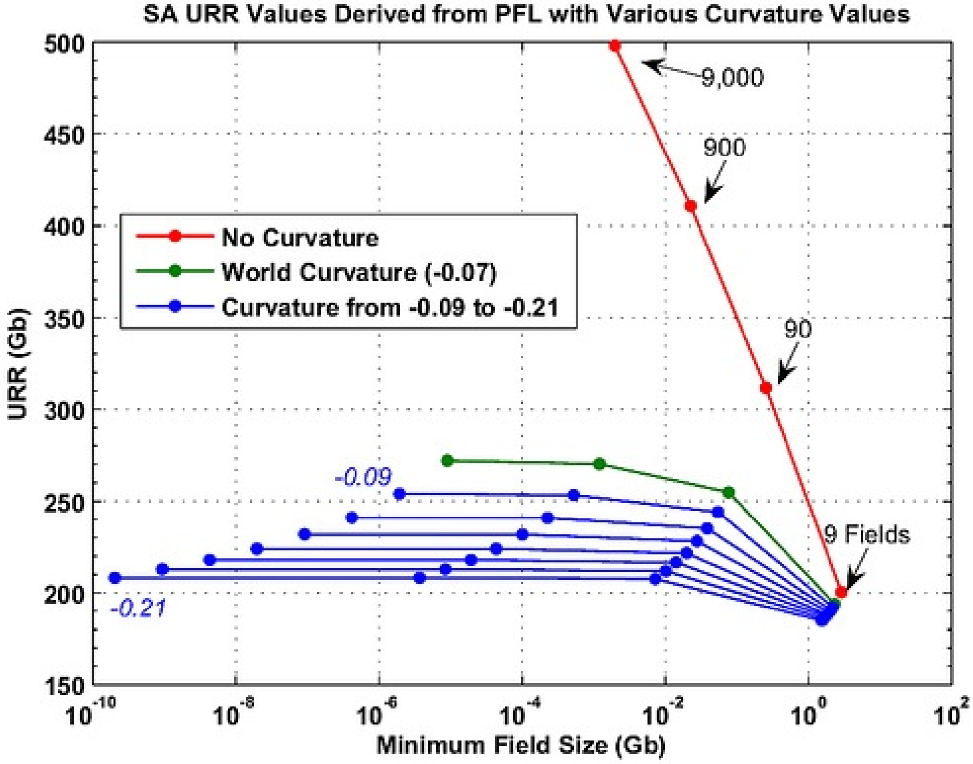

Fig. 3. Derived URR from the PFL shown on [Fig 2](). The URR value is function of the minimum oil field size considered.

If we fix an arbitrary field size cutoff value at 1 Mb, we get the URR values displayed on [Fig. 4](). We can see that using the world curvature at -*0.07* we get an URR at 270 Gb from about 2,000 fields which is remarkably close to the ASPO own estimate (275 Gb). The official URR at 368 Gb would imply a curvature closer to zero around -*0.025* with also a much higher number of fields. We compare also with the estimated URR values we get using the Hubbert Linearization technique on the production data (see [Fig. 5]()). The first fit (HL1) gives a rather low URR at 186 Gb which would imply a strong curvature beyond -*0.3* with a small number of fields (< 400).

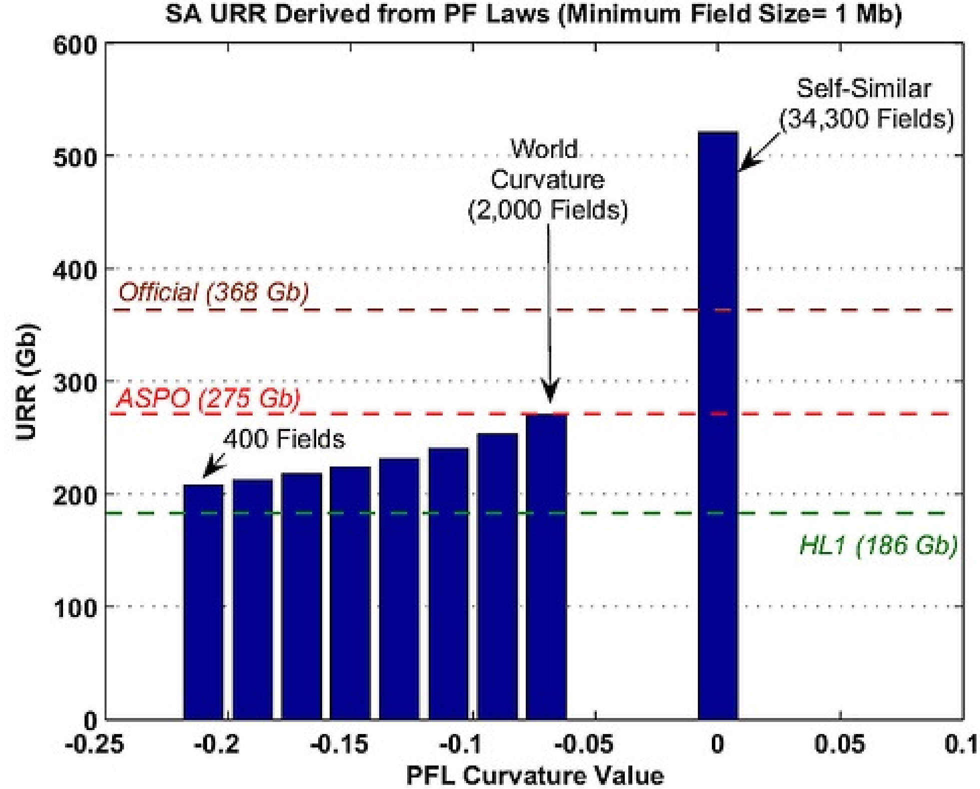

Fig. 4. Derived URR from [Fig. 3](#) by fixing the minimum oil field size at 1 Mb. HL1 and HL2 are the URR estimate from two different Hubbert Linearizations shown on [Fig. 5](#).

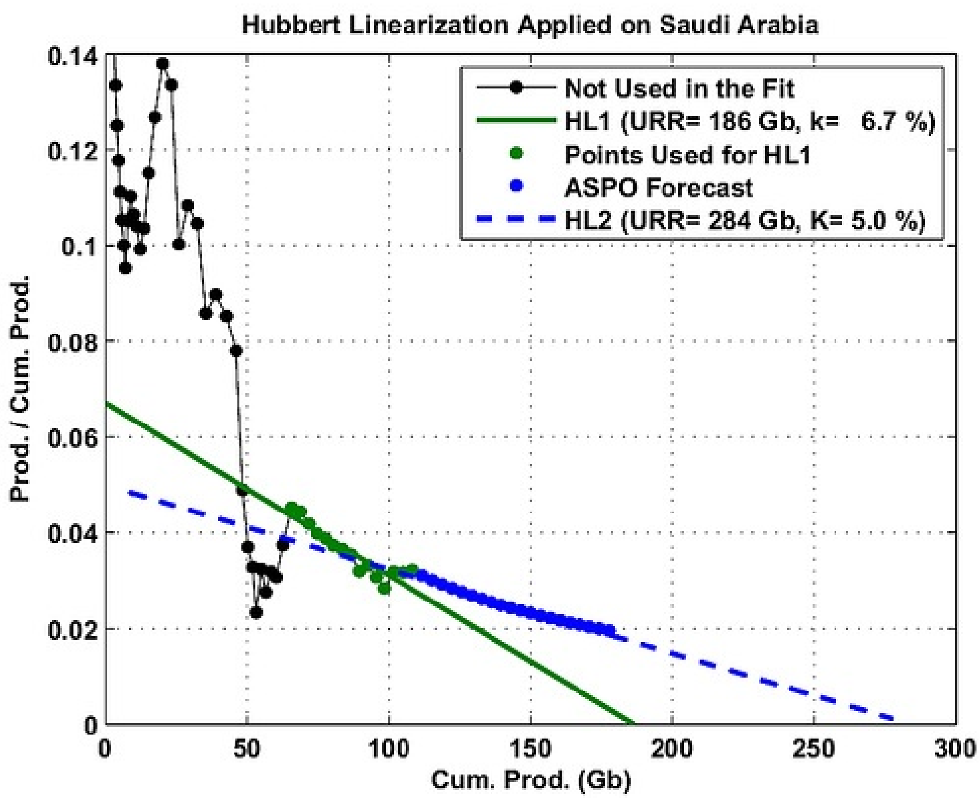

Fig. 5. Hubbert Linearizations on Saudi Arabia Production profile (data from BP, Crude + NGL). The blue points are the ASPO forecast which see a constant production level for the next 20 years at 9.5 mbpd ([newsletter 66](#)).

To further illustrate how hierarchical the oil production is, Fig. 6 gives the contribution of some oil field groups ranked according to their size.

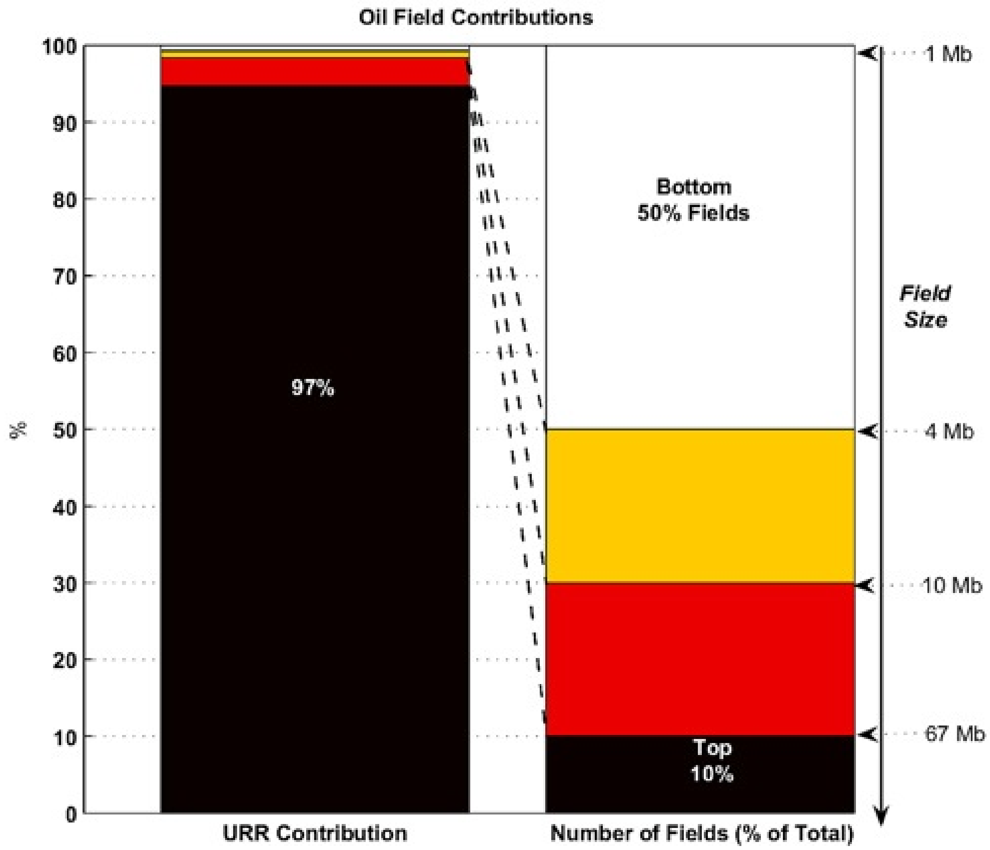

Fig. 6. Contributions from oil fields using the PFL law for Saudi Arabia with the world curvature (green line on Fig. 2 and Fig. 3). The top 10% of oil fields (size > 67 Mb) contributes to 97% of the total URR.

Discussion

The results are promising despite being based on partial and poor quality data. In particular, I find intriguing that the PFL will lead to a reasonable URR based on a curvature value derived from the world and the UK datasets. The eventual universality of this curvature value (*c=-0.07*) could be confirmed on other datasets such as Norway and the US. It is difficult to understand what is affecting the curvature value. My guess is that the population of small fields is probably less exploited and that less efficient recovery techniques are applied for obvious reasons. A few observations:

1. combined with the Hubbert Linearization technique, the PFL could be useful for tortuous production profiles from immature countries such as Saudi Arabia, Iraq and Iran.
2. only the top fields are necessary for the fit which is interesting because they are usually the most mature and the most documented. However, we implicitly assume that the discovery of large fields has peaked early in the production history and that no giant or super-giants will be discovered.
3. the PFL integrates naturally contributions from small fields and the derived URR is dependent on the minimum field size. Therefore, some reserve growth can be simulated by changing the small field cutoff value.

Further readings:

- Jean Laherrère: “Parabolic fractal” distributions in Nature. (in French but has many interesting graphs),
- Ada A. Adamic. Zipf, Power-laws, and Pareto - a ranking tutorial
- William J. Reed. The Pareto, Zipf and other power laws
- Narushige SHIODE. Power Law Distributions in Real and Virtual Worlds.