EROEI and Net Energy in the Exploitation of Natural Resources. A Study based on the Lotka-Volterra Model

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Abstract
Recently, it has been shown that the Lotka-Volterra model can be used for a quantitative description of the exploitation of non renewable resources in a free market economy [Bardi and Lavacchi 2009]. The present paper examines how the model describes the behavior of the system in terms of energy return for energy invested (EROEI) and Net Energy (energy returned minus energy expended). We show that these factors are correctly described by the model and that the evolution of the system leads to conditions in which an energy resource – such as crude oil – can be extracted in condition of negative net energy and EROEI smaller than 1.

1. Introduction

The "Hubbert Model" [Hubbert 1962] is an empirical model developed in the 1960s to describe the production of crude oil or – in general – of mineral resources. It is often observed that when a mineral resource is exploited in free market conditions, the extraction – or production – curve is bell shaped; nearly symmetric and with a maximum that is often termed "Hubbert peak" [Bardi and Pagani 2008]. It is generally believed that the Hubbert behaviour is related to the declining Energy return for energy invested (EROEI or EROI) of exploitation [Hall et al 2008, Murphy 2009]. That is, the decline in production is the result of the increasing costs of extraction. With lower profits, companies involved in extraction find themselves short of capital and must reduce investments. This process leads, eventually, to "peaking" of production and to its successive decline. This behaviour can also be observed for renewable resources which are exploited so fast that there is no time for the system to replenish the available stock. In this case, the term often used is "overexploitation" [Catton 1982] and has been observed, for instance, in the case of whale oil in 19th century [Bardi 2007]
In the present paper we will examine how a simple model can quantify the factors that lead to peaking and overexploitation. The model used here is based on the well known Lotka-Volterra (LV) model [Lotka 1925, Volterra 1926], also known as the "predator-prey" and "foxes and rabbits" model. Real biological systems are too complex to be captured by a

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simple two species model, but the LV model has found applications in economics, where it is sometimes known as “Free Access” model [Smith 1968]. In a recent study, Bardi and Lavacchi [2009] found that it is possible to use a simplified version of the model – one that doesn't account for the reproduction of the prey – to describe a number of cases of exploitation of non-renewable resources, such as crude oil, or of slowly renewable ones, such as whale oil. According to this study, this simplified LV model is equivalent to the Hubbert model. We will show how the LV model can provide a quantification of factors such as the EROEI and the Net Energy of the exploitation process and confirm that the Hubbert behavior is related to a declining energy yield.

2. EROEI and net energy.

Exploiting an energy resource can never been 100% efficient. For instance, if we consider oil wells, in order to exploit the chemical energy stored in the well, we must expend some energy in operations such as prospecting, drilling, extracting, processing, transporting etc. EROEI (Energy Return for Energy Invested) is defined as the ratio of the energy obtained from the resource to the energy expended in all production phases, including also the energy involved in the disposal of the equipment and installations involved [Hall et al. 2008, 2009]. A related concept is that of Net Energy, defined as the energy produced minus the energy expended in the whole process of exploitation. When the EROEI is equal to 1 or lower, the net energy is zero or lower. There exist also other concepts related to the energy efficiency of a process (e.g. “transformity”) which will not be considered here.

EROEI is a very useful concept for understanding the real value of a resource in economic and energy terms. Obviously, larger EROEIs are preferable and an EROEI smaller than one corresponds to a net loss of energy. However, some processes may be carried out even at low EROEIs, even smaller than one, as a result of specific choices of the economic system. As an example, biofuels, and in particular ethanol, have a low EROEI [Pimentel and Patzek 2005] but are produced using energy subsidies from higher EROEI fossil fuels.

Despite its usefulness, the definition of EROEI suffers of some uncertainty for two reasons: one is that energy quality is not taken explicitly into account, the second is that the boundaries of the system can be defined in different ways. For the first problem, energy quality, a better definition of EROEI would be in terms of exergy, that is the fraction of energy involved which able to do useful work, rather than simply energy. In practice, this is rarely a serious problem. The other uncertainty in the EROEI calculation, the boundaries of the system, is more complex. What factors should be exactly taken into account in defining the energy expended? For instance if the technicians involved in oil extraction take a plane to reach an oil field, the energy involved in the trip should obviously be accounted as a cost. But, if the same technicians take a vacation to Hawai'i, should the fuel energy of the trip be taken into account? It is customary to consider only some of these costs, but not others, according to norms defined in the “Life Cycle Analysis” (LCA) concept. These norms are defined in protocols such as, for instance, the ASTM E1991 – 05. If, however, one wants to take into account everything that is done with energy source (including, e.g. people's vacations) then we may speak of “full EROEI” or “societal EROEI.” [Hall et al. 2008, 2009]. The value of societal EROEI determines the surplus that can be utilized for all those activities that are considered part of what we call “civilization”, from cathedrals to poetry. The LCA-EROEI of specific processes will be normally larger than societal EROEI, because the latter includes more energy consuming processes.

3. The Lotka-Volterra model applied to resource exploitation.

Lotka [1925] and Volterra [1926] developed a well known model of "predator-prey" relationship in simple biological systems. The model was shown to be usable in economy, and in particular in the case of fisheries [Smith 1968]. In the implementation of the model developed for the exploitation of mineral resources [Bardi and Lavacchi, 2009] it was assumed that there are two main stock variables involved in the model: resources and capital. The amount of available resource is defined as the "resource stock," *R*. The other main variable of the model is the aggregate amount of economic resources being utilized in the exploitation; that is equipment, land, knowledge, human work, and similar. We called this aggregate amount "capital stock," *C*. *R'* and *C'* are defined as the flow (the variation as a function of time) of, respectively, resources and capital. Further parameters of the model are the initial stocks of resource (R_o)and of capital (C_o).
The resource (the "prey") can be extracted in proportion to the available capital (the "predator") and, at the same time, in proportion to the amount of the resource stock. This assumption is intuitively justified; the more equipment [e.g. oil rigs] is available, the higher the amount of resource that can be extracted/produced. On the other hand, there must be something to extract and the model assumes that the extraction rate will be proportional to the amount available. Implicitly, this assumption involves that resources are "graded" and that the "easy" (less expensive) resources are extracted [or produced] first.
The other fundamental assumption of the model is that capital is generated in an amount proportional to the amount of extracted resources. In other words, the resource stock is partly transformed into capital stock; let's say that the extracted oil is used to provide the energy necessary to build more oil rigs and other facilities. In more general terms, this transformation is generated via the sale of the resource on the market and the profits are used to create the equipment and facilities to produce more resource. Finally, we also assume that capital is dissipated over time.
These assumptions can be stated in mathematical form as two coupled differential equations derived from the Lotka-Volterra model. One term of the standard LV model is missing, that of the reproduction of the prey, that here is assumed not to occur or to occur very slowly.

1. $R' = - k_1CR$
2. $C' = k_2CR - k_3C$

This simplified model can describe a number of historical cases of resource exploitation, when the resource is non renewable (e.g. crude oil) or slowly renewable (e.g. whale oil) [Bardi and Lavacchi 2009].
In order for the model to be applicable, it is necessary to have data on at least some of the main parameters in the equations; e.g. historical data on production and on aggregate capital accumulation. These data may be measured in terms of "proxies"; e.g. using the tonnage of the whaling fleet as a proxy for the total capital available to the whaling industry. An example of the results obtained for the US-48 lower states historical data on crude oil production is shown in the figure 1. Here, the "discoveries" parameter is used as a proxy for resource production and the number of wildcats as a proxy for the aggregate capital used by the oil industry for prospecting in the area considered. Other cases for which the model was found to give a good or acceptable production were gold production in South Africa and in California, whale oil production in 19th century and crude oil production in Norway [Bardi and Lavacchi 2009]

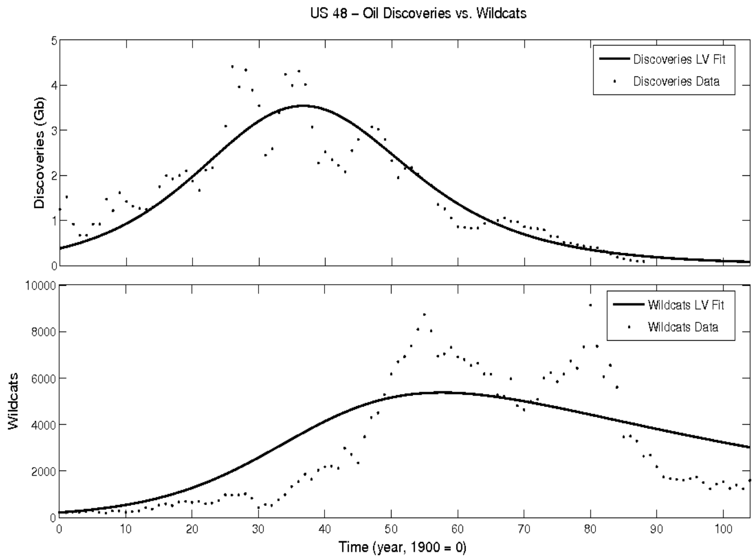

Figure 1. Fitting of the data for oil discovery in the US 48 lower states and of the number of wildcats. In this case, the number of wildcats is proportional to the capital used by the oil industry in the effort of discovering the resource (oil wells). From [Bardi and Lavacchi 2009]

4. EROEI and Net Energy in the Lotka-Volterra model

This Lotka-Volterra model – just as any model attempting to describe a physical system - must satisfy the laws of physics and, in particular, those of thermodynamics. It is clear that even in its simplest implementation, that of two biological species (foxes and rabbits), the two stocks involved are energy stocks. What the model describes is the flow of energy from one stock (rabbits) to another (foxes). The internal energy of the model is given by the sum of the stock of the resources and of capital. In the case of a non renewable resource, this total stock cannot increase, otherwise we would have creation of energy from nothing. There exist values of the constants of the model that provide such a creation and these values are forbidden by thermodynamics. In particular, if the stocks are measured using the same units, k_2 must be smaller than k_1 otherwise the transformation of rabbits into foxes would occur with an energy gain. If k_1=k_2, the transformation occurs at no metabolic cost: rabbits are not eaten, but hypnotized and convinced to be foxes.

Parameters such as EROEI and Net Energy are not explicitly expressed in the equations of the LV model, but can be calculated from the available parameters which, as mentioned before, include energy and energy flow. In a previous study [Bardi and Lavacchi 2009], we defined the "yield" of extraction as (R'/C), that is the ratio of production to capital. However, the relation of this parameter to the concept of EROEI needs to be discussed more in detail.

If we apply the LV model to an actual economic process, we are interested in defining parameters that measure its efficiency. In general, we are interested in maximizing production and minimizing costs and, in this case, we are interested in maximizing production and capital accumulation. The "costs" in the model are defined as the only term that produces an outflow (dissipation) of the accumulated capital. Therefore, we can define the economic yield of the process as the ratio of production (k_1RC) or capital accumulation (k_2RC) to capital dissipation (k_3C). The ratio of production to dissipation appears to be the factor most closely related to EROEI.

An example will clarify these considerations. Let's consider the common application of the LV model to a hypothetical two-species systems of foxes and rabbits (in this case, sterile rabbits). "Production" is the flow of captured rabbits (k_1RC). Capital accumulation is the number of newborn foxes (k_2RC). Capital dissipation can be seen as the energy spent by foxes in hunting rabbits. This parameter will be proportional to C (number of foxes) and will

actually be k_3C if foxes do nothing else in their life except hunting for rabbits and related activities. We need also to assume that foxes die of starvation but not by old age. This may be an approximately good view of the behavior of foxes and other animals.
Given these assumptions, the EROEI of the foxes/rabbits system can be defined as the ratio of the produced energy in terms of captured rabbits, k_1RC, to the expended energy for hunting (k_3C). If – as mentioned before - it is assumed that foxes do nothing in their life except reproducing and hunting rabbits then we arrive to the formula:

EROEI = k_1R/k_3

However, in an economic process such as oil extraction only a fraction of the profits from oil extraction will be re-invested in the same process. For instance, only part of the profits made by the oil industry will be re-invested in prospecting for oil and for developing oil fields and related facilities. We may consider that the definition given before, EROEI = k_1R/k_3, actually measures the "full EROEI" or "societal EROEI", and not just the extraction related EROEI. It will be therefore smaller than th conventional "LCA-EROEI. In any case, concept that EROEI is proportional to R (the resource stock) remains useful in the reasonable assumption that the fraction of profits re-invested by the industry in the exploitation of a resource remains approximately constant over the resource lifetime.
Note that there is no element in the formula that would stop processing when the EROEI becomes smaller than one; when that occurs, exploitation will continue utilizing previously accumulated energy resources. Note also that, since production is given by R', a maximum in the production curve will correspond to a flex in the EROEI curve.
From this result, we can proceed with the determination of the form for Net Energy (NE) which we may define as production minus dissipation. That is

$NE= C(k_1R - k_3)$

We see that for R sufficiently small, that is in the final stages of exploitation, the net energy of the system becomes negative and it remains so. We can also write this relation as:

$NE=CR(k_1-k_3/R)$

Assuming that R is relatively large, that is we are not at the final stages of the exploitation process, we can approximate NE as equal to k_1CR, that is to production. Therefore, we expect Net Energy to have a maximum that takes place, approximately, near the production peak.

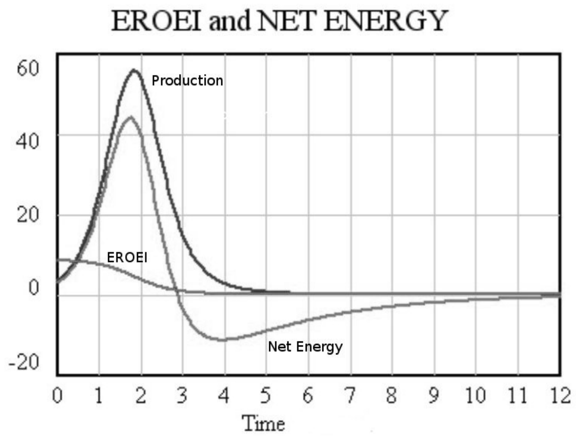

Figure 2. Qualitative solutions of the LV-system obtained using the Vensim software. The parameters reported are production, net energy and EROEI.

Qualitative simulations performed using the Vensim software confirm both this statement and the one that EROEI should have a flex in correspondence to the production peak; as shown here:
These calculations qualitatively correlate with the historical data on the EROEI of oil extraction from the US-48 lower states (Murphy 2009, Cleveland 2005) as shown in the following figure. Despite the small number of points available, it is possible that the flex in the EROEI curve occurs around 1970 and therefore it corresponds to the production peak in the region.

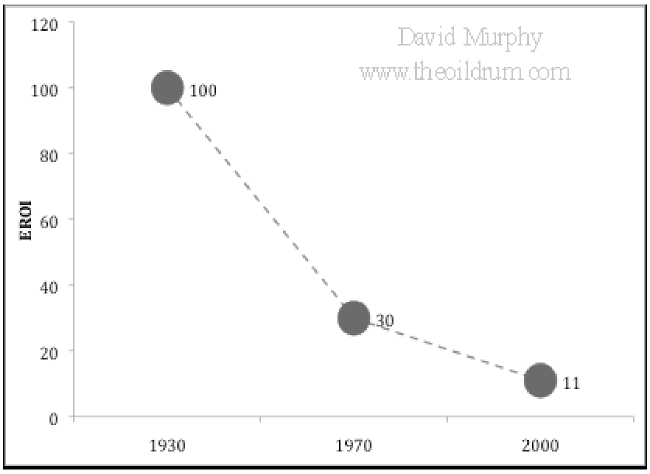

Figure 3. Historical EROEI of crude oil extraction in the lower 48 US states. Figure from [Murphy 2009]

These considerations raise the question of whether it is possible to use the Lotka-Volterra model to determine the EROEI instead of using the standard LCA analysis. There are two problems in this sense: the first is the use of proxy data for the fitting of the historical trends, the second is that EROEI, as defined within the model, implies that all the energy produced by the system is reused to produce more energy, an obviously non realistic assumption. Regarding the use of proxy data, it is possible to normalize the LV equations and to obtain the real EROEI as a function of the measured parameters. For the second problem, however, the LV model cannot say anything about the societal decision of which fraction of the profits from the exploitation of a resource have to be allocated to further exploitation. Indeed, tests made with the US-48 crude oil system studied in a previous paper [Bardi and Lavacchi 2009] show that the EROEI calculated from the LV model is much smaller than the LCA calculated EROEI [Cleveland 2005], as expected. On the basis of the available data, the “LV-EROEI” for crude oil in the US lower 48 states is calculated as around 4 at the start of the production cycle, whereas the LCA data indicate a value of about 100. This result may indicate that only about 5% of the energy produced by oil extraction was re-invested in oil extraction.
The results presented here have to be considered as preliminary and tentative, but we can conclude that the LV model offers useful insights on the mechanism of resource exploitation and it may offer in the future a route for modeling and understanding economic processes which are vital for society. More than its capability of quantification, the LV model offers a simple mental tool – a “mind sized model” [Papert 1980] to grasp the main elements of the overexploitation of natural resources (including the ability of the atmosphere to contain CO_2 without overheating) which is the main problem that our civilization is facing today.

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